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Polarization observables in elastic electron deuteron scattering including parity- and time-reversal–violating contributions

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Abstract. The general formalism for polarization observables in elastic electron deuteron scattering is extended to incorporate parity- and time-reversal–violating contributions. Parity-violating effects arise from the interference of γ and Z exchange as well as from the hadronic sector via a small parity-violating component in the deuteron. In addition we have allowed for time-reversal-invariance–violating contributions in the hadronic sector. Formal expressions for the additional structure functions are derived, and their decomposition into the various multipole contributions are given explicitly.

PACS. 11.30.Er Charge conjugation, parity, time reversal and other discrete symmetries – 24.70.+s Polarization phenomena in reactions – 24.80.+y Nuclear tests of fundamental interactions and symmetries – 25.30.Bf Elastic electron scattering

1 Introduction

The study of polarization observables in electroweak (e.w.) reactions is an important tool in order to investigate small but interesting dynamical effects, which normally are buried under the dominant amplitudes in unpolarized total and differential cross-sections, but which often may show up significantly in certain polarization observables. The reason for this feature lies in the fact that such small amplitudes or small contributions to large amplitudes may be amplified by interference with dominant amplitudes, or that dominant amplitudes interfere destructively leaving thus more room to the small amplitudes. For example, this fact has been exploited in elastic electron deuteron scattering in order to disentangle the charge quadrupole form factor from the monopole one by measuring the tensor asymmetry T_{20} or equivalently the tensor recoil polarization P_{20}. Other prominent examples are the measurement of parity violation of the e.w. interaction, and the study of T-noninvariant form factors in the same process.

A quite thorough discussion of polarization observables of elastic electron-deuteron scattering in the one-photon-approximation has been given by Gourdin and Piketty [1] and by Schildknecht [2] for the case of parity (P) and time reversal (T) invariant currents. The consequences of P-violating contributions from weak neutral currents on certain polarization observables for this process have been considered previously by several authors [3–6]. Furthermore, the influence of T-violation on the vector recoil polarization has been treated in [7–9]. However, it seems that no systematic formalism for polarization observables has been established for electroweak scattering including weak neutral currents arising from Z exchange. It is the aim of the present paper, to give a comprehensive and systematic derivation of all polarization observables for this reaction including parity- and time-reversal-invariance–violating contributions. To this end, we first review briefly in sect. 2 the basic ingredients for elastic electron scattering in the one-boson-exchange approximation. The general definition of a polarization observable is given in sect. 3, while explicit expressions in terms of structure functions and form factors are derived in sect. 4. Also the corresponding beam, target and beam-target asymmetries are given there. Various details are presented in several appendices.

2 Basic formalism

In this section we briefly present the basic formalism for elastic electron deuteron scattering in the one-boson-exchange approximation including Z exchange. The general expression for any observable, *i.e.*, cross-section and recoil polarization including the dependence on beam and target polarization, is given by

$$\mathcal{O}_X\, d\sigma_{fi} = (2\pi)^{-2}\delta^{(4)}(d'-q-d)\\ \times\mathrm{tr}(\mathcal{M}^\dagger_{fi}\,\widehat{O}_X\,\mathcal{M}_{fi}\hat\rho^e\hat\rho^d)\,\frac{m_e^2\,d^3k_2}{4k_{1,0}k_{2,0}}\,\frac{d^3d'}{2M_dE'_d}\,, \qquad (1)$$

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where the observable $\mathcal{O}_X$ is characterized by a subscript X, which refers to the various polarizations of the final deuteron state. It is represented by an appropriate operator $\hat{O}_X$ and will be specified later. The momenta of the initial and scattered electrons (mass m_e) are denoted by k_1 and k_2, respectively, and $q_\mu^2 = q_0^2 - \mathbf{q}^2$ the four-momentum transfer squared ($q = k_1 - k_2$). The initial and final deuteron momenta are denoted by $d = (E_d, \mathbf{d})$ and $d' = (E'_d, \mathbf{d}')$, respectively, and the deuteron mass by M_d. The density matrices $\hat{\rho}^e$ and $\hat{\rho}^d$ describe possible beam and target polarization. Covariant normalization has been assumed, *i.e.*, $(2\pi)^3 E/m$ for fermions and $(2\pi)^3 2E$ for bosons.

The amplitude $\mathcal{M}_{fi}$ contains in the lowest order, *i.e.*, in the one-boson-exchange approximation, contributions from both virtual γ and Z exchange with the latter naturally being strongly suppressed since we restrict ourselves to the low-momentum transfer region ($-q_\mu^2 \ll M_Z^2$). The invariant matrix element thus contains two contributions [10]

$$\mathcal{M}_{fi} = \frac{e^2}{q_\mu^2}\, j^{(\gamma)\,\mu} J^{(\gamma)}_{fi,\,\mu} + \sqrt{2}\widetilde{G}_{\mathrm{F}}\, j^{(Z)\,\mu} J^{(Z)}_{fi,\,\mu}\,. \tag{2}$$

Here and in the following, the superscripts γ and Z indicate the electromagnetic and weak neutral current contributions. The lepton and hadron currents are denoted by $j_\mu^{(\gamma/Z)}$ and $J^{(\gamma/Z)}_{fi,\,\mu}$, respectively. Furthermore, e denotes the elementary charge with $\alpha = e^2/4\pi$ as fine structure constant, and $\widetilde{G}_{\mathrm{F}}$ is related to the weak Fermi coupling constant G_{F} by

$$\widetilde{G}_{\mathrm{F}}(q_\mu^2) = \frac{M_Z^2}{M_Z^2 - q_\mu^2}\, G_{\mathrm{F}} = \frac{\sqrt{2}\, g^2}{8\cos^2\theta_{\mathrm{W}}\,(M_Z^2 - q_\mu^2)}\,, \tag{3}$$

where g denotes the electroweak coupling constant, θ_{W} the Weinberg angle, and $e = g\sin\theta_{\mathrm{W}}$.

The lepton currents are defined by

$$j^{(\gamma)\,\mu} = j^{(v)\,\mu}\,, \tag{4}$$

$$j^{(Z)\,\mu} = g_v^e\, j^{(v)\,\mu} + g_a^e\, j^{(a)\,\mu}\,, \tag{5}$$

where we have introduced the lepton vector and axial currents by

$$j^{(v)\,\mu} = \bar{u}(k_2)\,\gamma^\mu\, u(k_1)\,, \tag{6}$$

$$j^{(a)\,\mu} = \bar{u}(k_2)\,\gamma^\mu\gamma_5\, u(k_1)\,. \tag{7}$$

Furthermore, one has

$$g_v^e = -\frac{1}{2} + 2\sin^2\theta_{\mathrm{W}}\,, \tag{8}$$

$$g_a^e = \frac{1}{2}\,. \tag{9}$$

Note, that our expressions for the neutral currents contain an additional factor 1/2 compared to ref. [10]. The hadronic current J_μ is specified later. However, for formal reasons it is convenient to distinguish the contributions arising from the coupling to the lepton vector and axial currents by introducing

$$J_{fi,\,\mu}(\mathcal{V}) = J^{(\gamma)}_{fi,\,\mu} + J^{(Z^{\mathcal{V}})}_{fi,\,\mu}\,, \tag{10}$$

$$J_{fi,\,\mu}(\mathcal{A}) = J^{(Z^{\mathcal{A}})}_{fi,\,\mu}\,, \tag{11}$$

where

$$J^{(Z^{\mathcal{V}/\mathcal{A}})}_{fi,\,\mu} = \widetilde{G}_{v/a}\, J^{(Z)}_{fi,\,\mu} \tag{12}$$

with

$$\widetilde{G}_{v/a} = \sqrt{2}\, g^e_{v/a}\, \widetilde{G}_{\mathrm{F}}\, q_\mu^2\, e^{-2}\,. \tag{13}$$

We would like to emphasize, that the argument $\mathcal{V}$ and $\mathcal{A}$ merely indicates to which type of lepton current the hadronic current couples. Both hadronic currents, $J_{fi,\,\mu}(\mathcal{V})$ as well as $J_{fi,\,\mu}(\mathcal{A})$, contain vector and axial pieces (see below eqs. (32) and (33)). Then the invariant matrix element takes the form

$$\mathcal{M}_{fi} = \frac{e^2}{q_\mu^2}\Big(j^{(v)\,\mu}\, J_{fi,\,\mu}(\mathcal{V}) + j^{(a)\,\mu}\, J_{fi,\,\mu}(\mathcal{A})\Big)\,. \tag{14}$$

Allowing for longitudinal electron polarization of degree h, one then finds

$$\begin{aligned}
&\frac{m_e^2}{M_d^2}\,\mathrm{tr}(\mathcal{M}^\dagger_{fi}\,\widehat{O}_X\,\mathcal{M}_{fi}\hat{\rho}^e\hat{\rho}^d) = \\
&\left(\frac{e^2}{q_\mu^2}\right)^2\Big[\eta^{vv}_{\mu\nu}(h)\Big(W^{\mathcal{V}\mathcal{V},\,\mu\nu}_{fi}(\widehat{O}_X,\,\hat{\rho}^d) + W^{\mathcal{A}\mathcal{A},\,\mu\nu}_{fi}(\widehat{O}_X,\,\hat{\rho}^d)\Big) \\
&+\eta^{va}_{\mu\nu}(h)\Big(W^{\mathcal{V}\mathcal{A},\,\mu\nu}_{fi}(\widehat{O}_X,\,\hat{\rho}^d) + W^{\mathcal{A}\mathcal{V},\,\mu\nu}_{fi}(\widehat{O}_X,\,\hat{\rho}^d)\Big)\Big]\,,
\end{aligned} \tag{15}$$

where one has two types of lepton tensors $\eta^{vv}_{\mu\nu}$ and $\eta^{va}_{\mu\nu}$, where the latter arises from the interference of the lepton vector with the lepton axial current,

$$\eta^{vv}_{\mu\nu}(h) = \eta^0_{\mu\nu} + h\eta'_{\mu\nu}\,, \tag{16}$$

$$\eta^{va}_{\mu\nu}(h) = \eta'_{\mu\nu} + h\eta^0_{\mu\nu}\,. \tag{17}$$

In the high-energy limit, *i.e.*, electron mass $m_e = 0$, one has

$$\begin{aligned}
\eta^0_{\mu\nu} &= (k_{1\,\mu}k_{2\,\nu} + k_{2\,\mu}k_{1\,\nu}) - g_{\mu\nu}k_1\cdot k_2 \\
&= \frac{1}{2}(k_\mu k_\nu - q_\mu q_\nu + g_{\mu\nu}q_\rho^2)\,,
\end{aligned} \tag{18}$$

$$\begin{aligned}
\eta'_{\mu\nu} &= i\varepsilon_{\mu\nu\alpha\beta}k_2^\alpha k_1^\beta \\
&= \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta\,,
\end{aligned} \tag{19}$$

where $k = k_1 + k_2$. The hadronic tensors, appearing in (15), are defined by

$$W^{\mathcal{C}'\mathcal{C},\,\mu\nu}_{fi}(\widehat{O}_X,\,\hat{\rho}^d) = \frac{1}{M_d^2}\,\mathrm{tr}(J^{\mu\,*}_{fi}(\mathcal{C}')\,\widehat{O}_X\,J^\nu_{fi}(\mathcal{C})\hat{\rho}^d)\,, \tag{20}$$

where $\mathcal{C}',\,\mathcal{C} \in \{\mathcal{V},\mathcal{A}\}$, and the trace refers to the deuteron spin quantum numbers.

3 Definition of a general polarization observable

Proceeding as in the electromagnetic case by switching to the usual three-dimensional representation of the lepton tensors in terms of virtual boson density matrices, one obtains in analogy to the pure electromagnetic process the following expression for an observable:

$$\mathcal{O}_X \frac{d\sigma^{\gamma+Z}}{d\Omega_{k_2}^{\mathrm{lab}}} = \frac{2\alpha^2}{q_\mu^4}\Big(\frac{k_2^{\mathrm{lab}}}{k_1^{\mathrm{lab}}}\Big)^2 \sum_{\lambda,\lambda'}\sum_{m',m,n',n} \rho^d_{mn}\Big[(\rho^0_{\lambda\lambda'} + h\rho'_{\lambda\lambda'}) \sum_{\mathcal{C}\in\{\mathcal{V},\mathcal{A}\}} t^*_{n'\lambda'n}(\mathcal{C})(\widehat{O}_X)_{n'm'}t_{m'\lambda m}(\mathcal{C}) + (h\rho^0_{\lambda\lambda'}+\rho'_{\lambda\lambda'}) \sum_{\mathcal{C}'\neq\mathcal{C}\in\{\mathcal{V},\mathcal{A}\}} t^*_{n'\lambda'n}(\mathcal{C}')(\widehat{O}_X)_{n'm'}t_{m'\lambda m}(\mathcal{C})\Big]. \quad (21)$$

Here, we have introduced the t-matrices, which are related to the various current matrix elements between the intrinsic deuteron states by

$$t_{m'\lambda m}(\mathcal{C}) = \frac{\sqrt{E'_d E_d}}{M_d}\,\langle m'|J_\lambda(\mathcal{C})|m\rangle\,. \quad (22)$$

The current components refer to a coordinate system with z-axis along $\mathbf{q}$, y-axis along $\mathbf{k}_1\times\mathbf{k}_2$, *i.e.*, perpendicular to the scattering plane, and x-axis chosen as to form a right-handed system, *i.e.*, $\hat{x} = \hat{y}\times\hat{z}$. Also the deuteron spin states refer to this system with $\mathbf{q}$ as quantization axis. Thus $\lambda = \pm 1$ refers to the transverse current components (with respect to $\mathbf{q}$), while the $\lambda = 0$ component is given by a combination of charge and longitudinal current component

$$\begin{aligned} J_0 &= -\frac{|\mathbf{q}|^2}{q_\mu^2}(\rho - \frac{\omega}{|\mathbf{q}|^2}\mathbf{q}\cdot\mathbf{J}) \\ &= \rho - \frac{\omega}{q_\mu^2}(\omega\rho - \mathbf{q}\cdot\mathbf{J})\,, \end{aligned} \quad (23)$$

which reduces to the charge density ρ for a conserved current. Furthermore, E_d and E'_d denote the initial and final deuteron energies, respectively. The c.m. motion of the initial and final deuteron states with c.m. momenta $\mathbf{d}$ and $\mathbf{d}'$, respectively, has been eliminated and we have switched to noncovariant normalization.

The spherical components of the two types of virtual boson density matrices obey the symmetry relations

$$\rho^{0/\prime}_{\lambda\lambda'} = \rho^{0/\prime}_{\lambda'\lambda}\,, \quad (24)$$

$$\rho^0_{-\lambda-\lambda'} = (-)^{\lambda+\lambda'}\rho^0_{\lambda\lambda'}\,, \quad (25)$$

$$\rho'_{-\lambda-\lambda'} = (-)^{\lambda+\lambda'+1}\rho'_{\lambda\lambda'}\,. \quad (26)$$

Here, $\rho^{0/\prime}$ can be expanded into independent components with respect to diagonal longitudinal (L) and transverse (T) contributions, and interference terms (LT and TT)

$$\rho^{0/\prime}_{\lambda\lambda'} = \sum_{\alpha\in\{L,T,LT,TT\}} \delta^{(\prime)\,\alpha}_{\lambda\lambda'}\,\rho^{(\prime)}_\alpha\,, \quad (27)$$

with

$$\begin{aligned} &\delta^L_{\lambda\lambda'} = \delta_{\lambda\lambda'}\delta_{\lambda 0}\,, && \delta^{LT}_{\lambda\lambda'} = \lambda'\delta_{\lambda 0} + \lambda\delta_{\lambda' 0}\,, \\ &\delta^T_{\lambda\lambda'} = \delta_{\lambda\lambda'}|\lambda|\,, && \delta^{TT}_{\lambda\lambda'} = \delta_{\lambda,\,-\lambda'}|\lambda|\,, \\ &\delta'^{L}_{\lambda\lambda'} = 0\,, && \delta'^{LT}_{\lambda\lambda'} = |\lambda'|\delta_{\lambda 0} + |\lambda|\delta_{\lambda' 0}\,, \\ &\delta'^{T}_{\lambda\lambda'} = \delta_{\lambda\lambda'}\lambda\,, && \delta'^{TT}_{\lambda\lambda'} = 0\,. \end{aligned} \quad (28)$$

The nonvanishing components are

$$\begin{aligned} &\rho_L = \rho^0_{00} = -\beta^2 q_\nu^2 \tfrac{\xi^2}{2\zeta}\,, && \rho_T = \rho^0_{11} = -\tfrac{1}{2}q_\nu^2\Big(1+\tfrac{\xi}{2\zeta}\Big)\,, \\ &\rho_{LT} = \rho^0_{01} = -\beta q_\nu^2 \tfrac{\xi}{\zeta}\sqrt{\tfrac{\zeta+\xi}{8}}\,, && \rho_{TT} = \rho^0_{-11} = q_\nu^2\tfrac{\xi}{4\zeta}\,, \\ &\rho'_{LT} = \rho'_{01} = -\tfrac{1}{2}\,\beta\tfrac{q_\nu^2}{\sqrt{2\zeta}}\,\xi\,, && \rho'_T = \rho'_{11} = -\tfrac{1}{2}q_\nu^2\sqrt{\tfrac{\zeta+\xi}{\zeta}}\,, \end{aligned} \quad (29)$$

with

$$\beta = \frac{|\mathbf{q}^{\mathrm{lab}}|}{|\mathbf{q}^{\,c}|}\,,\quad \xi = -\frac{q_\nu^2}{|\mathbf{q}^{\mathrm{lab}}|^2}\,,\quad \zeta = \tan^2\frac{\theta_e}{2}\,, \quad (30)$$

where β expresses the boost from the lab system to the frame in which the hadronic tensor is evaluated and $\mathbf{q}^{\,c}$ denotes the momentum transfer in this frame. In order to make contact to the kinematic functions $v_{\alpha^{(\prime)}}$ in the review of Musolf *et al.* [12], we note the simple relation (for $\beta = 1$)

$$\rho^{(\prime)}_\alpha = -\frac{q_\mu^2}{2\zeta}\,v_{\alpha^{(\prime)}}\,, \quad (31)$$

where $\alpha \in \{L,\, T,\, LT,\, TT\}$.

Now we will discuss the various hadronic tensors of (21) in detail. The hadronic currents can be classified according to their vector and axial current contributions. The e.m. current contains only a vector piece $J^\gamma_{fi,\,\mu}$ while the neutral current consists of both, vector and axial parts, $J^{Z_v}_{fi,\,\mu}$ and $J^{Z_a}_{fi,\,\mu}$, respectively. Thus for the hadron current interacting with the lepton vector current $J_{fi,\,\mu}(\mathcal{V})$ one has $J^{\gamma/Z^{\mathcal{V}}_v}_{fi,\,\mu}$ as vector part and $J^{Z^{\mathcal{V}}_a}_{fi,\,\mu}$ as axial part, *i.e.*,

$$\begin{aligned} J_{fi,\,\mu}(\mathcal{V}) &= J^\gamma_{fi,\,\mu} + \widetilde{G}_v\,(J^{Z_v}_{fi,\,\mu} + J^{Z_a}_{fi,\,\mu}) \\ &= J^\gamma_{fi,\,\mu} + J^{Z^{\mathcal{V}}_v}_{fi,\,\mu} + J^{Z^{\mathcal{V}}_a}_{fi,\,\mu}\,. \end{aligned} \quad (32)$$

The corresponding contributions to the hadron current $J_{fi,\,\mu}(\mathcal{A})$ interacting with the lepton axial current are $J^{Z^{\mathcal{A}}_v}_{fi,\,\mu}$ and $J^{Z^{\mathcal{A}}_a}_{fi,\,\mu}$, respectively,

$$\begin{aligned} J_{fi,\,\mu}(\mathcal{A}) &= \widetilde{G}_a\,(J^{Z_v}_{fi,\,\mu} + J^{Z_a}_{fi,\,\mu}) \\ &= J^{Z^{\mathcal{A}}_v}_{fi,\,\mu} + J^{Z^{\mathcal{A}}_a}_{fi,\,\mu}\,. \end{aligned} \quad (33)$$

Note, that $J^{Z^{\mathcal{V}}_{v/a}}_{fi,\,\mu}$ and $J^{Z^{\mathcal{A}}_{v/a}}_{fi,\,\mu}$ are related by the ratio of g^e_v/g^e_a, *i.e.*,

$$J^{Z^{\mathcal{V}}_v}_{fi,\,\mu} = g^e_v/g^e_a\,J^{Z^{\mathcal{A}}_v}_{fi,\,\mu} \quad\text{and}\quad J^{Z^{\mathcal{V}}_a}_{fi,\,\mu} = g^e_v/g^e_a\,J^{Z^{\mathcal{A}}_a}_{fi,\,\mu}\,. \quad (34)$$

Thus $J^{Z^{\mathcal{V}}_{v/a}}_{fi,\mu}$ will be suppressed compared to $J^{Z^{\mathcal{A}}_{v/a}}_{fi,\mu}$. Since we allow also for parity violation in the hadronic states, any current matrix element can be split into two contributions with opposite parity transformation properties, *i.e.*,

$$J^{c}_{fi} = J^{c_{pc}}_{fi} + J^{c_{pnc}}_{fi}\,, \qquad (35)$$

where, denoting the dominant component by an upper index "*pc*" and the small, parity-violating component of opposite parity by "*pnc*"

$$J^{c_{pc}}_{fi} = {}_{pc}\langle f|J|i\rangle_{pc} + {}_{pnc}\langle f|J|i\rangle_{pnc}\,, \qquad (36)$$
$$J^{c_{pnc}}_{fi} = {}_{pnc}\langle f|J|i\rangle_{pc} + {}_{pc}\langle f|J|i\rangle_{pnc}\,, \qquad (37)$$

where $|\rangle_{pc}$ denotes the dominant parity-conserving wave function component and $|\rangle_{pnc}$ the small parity-violating component. Thus, in order to classify the various contributions, we will define two symbolic index sets $\mathcal{C}_{\mathcal{V}}$ and $\mathcal{C}_{\mathcal{A}}$ according to the interaction with the lepton vector and axial currents, respectively, by

$$\mathcal{C}_{\mathcal{V}} = \{\gamma_{pc},\, \gamma_{pnc},\, Z^{\mathcal{V}}_{v,\,pc},\, Z^{\mathcal{V}}_{v,\,pnc},\, Z^{\mathcal{V}}_{a,\,pc},\, Z^{\mathcal{V}}_{a,\,pnc}\}\,, \qquad (38)$$
$$\mathcal{C}_{\mathcal{A}} = \{Z^{\mathcal{A}}_{v,\,pc},\, Z^{\mathcal{A}}_{v,\,pnc},\, Z^{\mathcal{A}}_{a,\,pc},\, Z^{\mathcal{A}}_{a,\,pnc}\}\,. \qquad (39)$$

It is also convenient to introduce two other sets of current contributions according to their behaviour under parity transformations, whether they transform like a vector or like an axial current. They are defined by

$$\mathcal{C}_{pc} = \{\gamma_{pc},\, Z^{\mathcal{V}}_{v,\,pc},\, Z^{\mathcal{V}}_{a,\,pnc},\, Z^{\mathcal{A}}_{v,\,pc},\, Z^{\mathcal{A}}_{a,\,pnc}\}\,, \qquad (40)$$
$$\mathcal{C}_{pnc} = \{\gamma_{pnc},\, Z^{\mathcal{V}}_{v,\,pnc},\, Z^{\mathcal{V}}_{a,\,pc},\, Z^{\mathcal{A}}_{v,\,pnc},\, Z^{\mathcal{A}}_{a,\,pc}\}\,. \qquad (41)$$

In order to characterize the opposite behaviour with respect to parity, we will introduce a symbolic δ-function by

$$\delta^{P}_{c} := \left\{ \begin{array}{l} 0 \;\text{ for } c \in \mathcal{C}_{pc} \\ 1 \text{ for } c \in \mathcal{C}_{pnc} \end{array} \right\}. \qquad (42)$$

Furthermore, in order to be more general we will also allow for violation of time reversal invariance. Consequently, we will split each of the two sets $\mathcal{C}_{pc}$ and $\mathcal{C}_{pnc}$ into two subsets, one containing the contributions which respect time reversal invariance and the other those violating it, labeled in addition by "*tc*" and "*tnc*", respectively,

$$\mathcal{C}_{pc} = \mathcal{C}_{pc,\,tc} \cup \mathcal{C}_{pc,\,tnc}\,, \qquad (43)$$
$$\mathcal{C}_{pnc} = \mathcal{C}_{pnc,\,tc} \cup \mathcal{C}_{pnc,\,tnc}\,, \qquad (44)$$

where the four different sets are given by

$$\mathcal{C}_{pc,\,tc} = \{\gamma_{pc,\,tc},\, Z^{\mathcal{V}}_{v,\,pc,\,tc},\, Z^{\mathcal{V}}_{a,\,pnc,\,tc},\, Z^{\mathcal{A}}_{v,\,pc,\,tc},\, Z^{\mathcal{A}}_{a,\,pnc,\,tc}\}\,, \qquad (45)$$
$$\mathcal{C}_{pnc,\,tc} = \{\gamma_{pnc,\,tc},\, Z^{\mathcal{V}}_{v,\,pnc,\,tc},\, Z^{\mathcal{V}}_{a,\,pc,\,tc},\, Z^{\mathcal{A}}_{v,\,pnc,\,tc},\, Z^{\mathcal{A}}_{a,\,pc,\,tc}\}\,, \qquad (46)$$
$$\mathcal{C}_{pc,\,tnc} = \{\gamma_{pc,\,tnc},\, Z^{\mathcal{V}}_{v,\,pc,\,tnc},\, Z^{\mathcal{V}}_{a,\,pnc,\,tnc},\, Z^{\mathcal{A}}_{v,\,pc,\,tnc},\, Z^{\mathcal{A}}_{a,\,pnc,\,tc}\}, \qquad (47)$$
$$\mathcal{C}_{pnc,\,tnc} = \{\gamma_{pnc,\,tnc}, Z^{\mathcal{V}}_{v,\,pnc,\,tnc}, Z^{\mathcal{V}}_{a,\,pc,\,tnc}, Z^{\mathcal{A}}_{v,\,pnc,\,tnc}, Z^{\mathcal{A}}_{a,\,pc,\,tnc}\}. \qquad (48)$$

Correspondingly, in order to characterize the opposite transformation behaviour under time reversal we introduce

$$\delta^{T}_{c} := \left\{ \begin{array}{l} 0 \;\text{ for } c \in \mathcal{C}_{pc,\,tc} \cup \mathcal{C}_{pnc,\,tc} \\ 1 \text{ for } c \in \mathcal{C}_{pc,\,tnc} \cup \mathcal{C}_{pnc,\,tnc} \end{array} \right\}. \qquad (49)$$

As a shorthand, we will use

$$\delta^{PT}_{c} = \delta^{P}_{c} + \delta^{T}_{c}\,. \qquad (50)$$

Now we write the t-matrix element of (22) as a sum of the various current contributions labeled by a superscript "*c*"

$$t_{m'\lambda m}(\mathcal{V}/\mathcal{A}) = \sum_{c\in\mathcal{C}_{\mathcal{V}/\mathcal{A}}} t^{c}_{m'\lambda m}\,, \qquad (51)$$

and obtain for the hadronic current tensors in (21)

$$\sum_{\mathcal{C}\in\{\mathcal{V},\mathcal{A}\}} t^{*}_{n'\lambda n}(\mathcal{C})\, t_{m'\lambda' m}(\mathcal{C}) = \sum_{\mathcal{C}=(\mathcal{C}_{\mathcal{V}},\,\mathcal{C}_{\mathcal{A}})}\ \sum_{c',\,c\in\mathcal{C}} t^{c'\,*}_{n'\lambda' n}\, t^{c}_{m'\lambda m}\,,$$
$$\sum_{\mathcal{C}'\neq\mathcal{C}\in\{\mathcal{V},\mathcal{A}\}} t^{*}_{n'\lambda n}(\mathcal{C}')\, t_{m'\lambda' m}(\mathcal{C}) = \sum_{c'\in\mathcal{C}_{\mathcal{V}}}\ \sum_{c\in\mathcal{C}_{\mathcal{A}}} \Big(t^{c'\,*}_{n'\lambda' n}\, t^{c}_{m'\lambda m} + (c\leftrightarrow c')\Big)\,. \qquad (52)$$

Any of these current matrix elements $t^{c}_{m'\lambda m}$ can be expanded into multipoles

$$t^{c}_{m'\lambda m} = (-)^{\lambda}\, a_{\lambda}\, \frac{\sqrt{E'_d E_d}}{M_d} \sum_{L} i^{L} \hat{L} \langle 1m'|\mathcal{O}^{\lambda}_{L\lambda}(c)|1m\rangle = (-)^{1-m'+\lambda}\, a_{\lambda} \sum_{L} i^{L} \hat{L} \begin{pmatrix} 1 & L & 1 \\ -m' & \lambda & m \end{pmatrix} \mathcal{O}^{\lambda}_{L}(c)\,, \qquad (53)$$

where $a_{\lambda} = \sqrt{2\pi(1+\delta_{\lambda 0})}$, and

$$\mathcal{O}^{\lambda}_{LM} = \delta_{\lambda 0}\, \mathcal{C}_{LM} + \delta_{|\lambda| 1}\, (\mathcal{E}_{LM} + \lambda\, \mathcal{M}_{LM}) \qquad (54)$$

denotes a general multipole. The argument "*c*" of the multipole $\mathcal{O}^{\lambda}_{L}(c)$ in (53) indicates the current contribution. In (53) we have chosen the direction of the momentum transfer $\mathbf{q}$ as quantization axis for the deuteron spin states and have introduced for the reduced matrix elements of the multipole operators betwee the deuteron states the notation

$$\mathcal{O}^{\lambda}_{L}(c) = \frac{\sqrt{E'_d E_d}}{M_d}\, \langle 1\|\mathcal{O}^{\lambda}_{L}(c)\|1\rangle = \delta_{\lambda 0}\, C_L(c) + i\, \delta_{|\lambda| 1}\, (E_L(c) + \lambda\, M_L(c))\,. \qquad (55)$$

Here the factor $\sqrt{E'_d E_d}/M_d$ has been included for convenience in the definition of the reduced charge ($C_L(c)$) and transverse ($E_L(c)$, $M_L(c)$) matrix elements. Furthermore, a factor "*i*" has been separated from the transverse multipoles in order to have E_L and M_L as real quantities, because one has $(\mathcal{O}_L^\lambda(c))^* = (-)^\lambda \mathcal{O}_L^\lambda(c)$ (see Appendix A). From time reversal one has the following selection rules for the multipoles

$$C_L(c) : (-)^{L+\delta_c^T} = 1\,, \quad E/M_L(c) : (-)^{L+\delta_c^T} = -1\,. \quad (56)$$

On the other hand, the parity transformation yields as selection rules

$$(C/E)_L(c) : (-)^{L+\delta_c^P} = 1, \quad M_L(c) : (-)^{L+\delta_c^P} = -1\,. \quad (57)$$

Combining these selection rules, one finds as nonvanishing multipole contributions

$$\begin{array}{lll} C_0(c),\, C_2(c),\, M_1(c) & \text{for} & c \in \mathcal{C}_{pc,\,tc}\,,\\ E_2(c) & \text{for} & c \in \mathcal{C}_{pc,\,tnc}\,,\\ E_1(c) & \text{for} & c \in \mathcal{C}_{pnc,\,tc}\,,\\ C_1(c),\, M_2(c) & \text{for} & c \in \mathcal{C}_{pnc,\,tnc}\,. \end{array} \quad (58)$$

Before proceeding further, we have to specify the observable X in (1) describing any observable for the analysis of the final target spin state. We choose the representation $X = (IM\pm)$ ($I = 0, 1, 2$, $M \geq 0$) with a corresponding Hermitian operator in deuteron spin space

$$\widehat{O}_{IM\mathrm{sig}_M} = c_{M\mathrm{sig}_M}(\tau_M^{[I]} + \mathrm{sig}_M(-)^M \tau_{-M}^{[I]})\,, \quad (59)$$

with

$$c_{M\mathrm{sig}_M} = \begin{cases} \frac{1}{1+\delta_{M0}} & \text{for } \mathrm{sig}_M = +\,,\\ i & \text{for } \mathrm{sig}_M = -\,. \end{cases} \quad (60)$$

Here we have introduced a sign function by $\mathrm{sig}_M := \pm$, where the subscript M merely indicates to which variable it refers. One should note, that obviously for $(IM\mathrm{sig}_M) = (I0-)$ the operator vanishes, *i.e.*, $\widehat{O}_{I0-} = 0$.

The irreducible tensors $\tau^{[I]}$ are the usual statistical tensors for the parametrization of the density matrix of a spin-one particle

$$\rho^d = \frac{1}{3}\sum_{I=0}^{2}\sum_{M=-I}^{I} \tau_M^{[I]}\, P_{IM}^{d\,*}\,, \quad (61)$$

where P_{IM}^d characterizes the initial state polarization with $P_{00}^d = 1$.

The tensors $\tau^{[I]}$ are normalized as $\langle 1||\tau^{[I]}||1\rangle = \sqrt{3}\,\widehat{I}$, where $\widehat{I} = \sqrt{2I+1}$, *i.e.*, in detail

$$\tau^{[I]} = \begin{cases} \mathbb{1}_3 & \text{no polarization,}\\ \sqrt{\frac{3}{2}}\,S^{[1]} & \text{vector polarization,}\\ \sqrt{3}\,S^{[2]} & \text{tensor polarization,} \end{cases} \quad (62)$$

where $\mathbb{1}_3$ is the unit matrix, $S^{[1]}$ the spin-one operator, and $S^{[2]} = [S^{[1]} \times S^{[1]}]^{[2]}$ the tensor operator whose Cartesian components are

$$S_{kl}^{[2]} = \frac{1}{2}(S_k S_l + S_l S_k) - \frac{2}{3}\,\delta_{kl}\,. \quad (63)$$

Using the relation of the $\widehat{O}_{IM\mathrm{sig}_M}$ to the Cartesian spin operators

$$\begin{array}{ll} S_{x/y} = \mp\frac{1}{\sqrt{3}}\widehat{O}_{11\pm}\,, & S_z = \sqrt{\frac{2}{3}}\widehat{O}_{10+}\,,\\ S_{xx/yy}^{[2]} = \pm\frac{1}{2\sqrt{3}}\widehat{O}_{22+} - \frac{1}{3\sqrt{2}}\widehat{O}_{20+}\,, & S_{zz}^{[2]} = \frac{\sqrt{2}}{3}\,\widehat{O}_{20+}\,,\\ S_{xy}^{[2]} = -\frac{1}{2\sqrt{3}}\widehat{O}_{22-}\,, & S_{zx/zy}^{[2]} = \mp\frac{1}{2\sqrt{3}}\widehat{O}_{21\pm}\,, \end{array} \quad (64)$$

one finds for the relation of the above defined observables $\mathcal{O}_{IM\mathrm{sig}_M}$ to the Cartesian spin observables P_k and P_{kl}

$$\begin{array}{ll} P_{x/y} = \mp\frac{1}{\sqrt{3}}\mathcal{O}_{11\pm}\,, & P_z = \sqrt{\frac{2}{3}}\mathcal{O}_{10+}\,,\\ P_{xx/yy} = \pm\frac{1}{2\sqrt{3}}\mathcal{O}_{22+} - \frac{1}{3\sqrt{2}}\mathcal{O}_{20+}\,, & P_{zz} = \frac{\sqrt{2}}{3}\,\mathcal{O}_{20+}\,,\\ P_{xy} = -\frac{1}{2\sqrt{3}}\mathcal{O}_{22-}\,, & P_{zx/zy} = \mp\frac{1}{2\sqrt{3}}\mathcal{O}_{21\pm}\,, \end{array} \quad (65)$$

where the Cartesian observables are defined by the deuteron density matrix in the form

$$\rho^d = \frac{1}{3}\,(\mathbb{1}_3 + \mathbf{P}\cdot\mathbf{S} + \sum_{kl} P_{kl} S_{kl}^{[2]})\,. \quad (66)$$

From now on we will assume that the density matrix is diagonal with respect to a certain orientation axis, characterized by spherical angles θ_d and ϕ_d. Then one can write

$$P_{IM}^d = P_I^d e^{iM\phi_d} d_{M0}^I(\theta_d)\,, \quad (67)$$

with the deuteron vector (P_1^d) and tensor (P_2^d) polarization parameters which are related to the occupation probabilities p_m of the different spin projection states of the deuteron with respect to the orientation axis as quantization axis by

$$P_1^d = P_{10}^d = \sqrt{\frac{3}{2}}(p_1 - p_{-1})\,, \quad (68)$$

$$P_2^d = P_{20}^d = \frac{1}{\sqrt{2}}\,(3(p_1 + p_{-1}) - 2)\,. \quad (69)$$

4 Structure functions and asymmetries

Inserting these various expressions into (21), one obtains finally for an observable $X = (I'M'\mathrm{sig}_{M'})$ in terms of four types of structure functions $F_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X)$ and $\widetilde{F}_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X)$

$$\begin{aligned} \mathcal{O}_X \frac{d\sigma^{\gamma+Z}}{d\Omega_{k_2}^{\mathrm{lab}}} = {}& \\ \sigma_{\mathrm{Mott}} \sum_{I=0}^{2} P_I^d \sum_{M=0}^{I} & \sum_{\mathrm{sig}_M=\pm} \cos\Big(M\phi_d + \frac{\pi}{4}(1-\mathrm{sig}_M 1)\Big) d_{M0}^I(\theta_d)\\ \times \sum_{\alpha\in\{L,\,T,\,LT,\,TT\}} & \Big[v_\alpha\Big(F_\alpha^{IM\mathrm{sig}_M}(X) + h\widetilde{F}_\alpha^{IM\mathrm{sig}_M}(X)\Big)\\ & + v'_\alpha\Big(hF_\alpha'^{\,IM\mathrm{sig}_M}(X) + \widetilde{F}_\alpha'^{\,IM\mathrm{sig}_M}(X)\Big)\Big]\,, \end{aligned} \quad (70)$$

where we have introduced the Mott cross-section

$$\sigma_{\mathrm{Mott}} = \frac{\alpha^2 \cos^2 \frac{\theta_e^{\mathrm{lab}}}{2}}{4 \sin^4 \frac{\theta_e^{\mathrm{lab}}}{2}} \frac{k_2^{\mathrm{lab}}}{(k_1^{\mathrm{lab}})^3}\,, \tag{71}$$

and switched to the $v_\alpha^{(\prime)}$'s instead of the $\rho_\alpha^{(\prime)}$'s according to (31). Their explicit form is

$$\begin{aligned}
v_L &= \tfrac{\beta^2}{(1+\eta)^2}\,,\\
v_T &= \tfrac{1}{2\,(1+\eta)} + \tan^2 \tfrac{\theta_e^{\mathrm{lab}}}{2}\,,\\
v_{LT} &= \tfrac{1}{\sqrt{2}}\, \tfrac{\beta}{1+\eta}\, \sec \tfrac{\theta_e^{\mathrm{lab}}}{2} \sqrt{\tfrac{1+\eta \sin^2 \frac{\theta_e^{\mathrm{lab}}}{2}}{1+\eta}}\,,\\
v_{TT} &= -\tfrac{1}{2\,(1+\eta)}\,,\\
v'_{LT} &= \tfrac{1}{\sqrt{2}}\, \tfrac{\beta}{1+\eta}\, \tan \tfrac{\theta_e^{\mathrm{lab}}}{2}\,,\\
v'_T &= \sec \tfrac{\theta_e^{\mathrm{lab}}}{2}\, \tan \tfrac{\theta_e^{\mathrm{lab}}}{2} \sqrt{\tfrac{1+\eta \sin^2 \frac{\theta_e^{\mathrm{lab}}}{2}}{1+\eta}}\,,
\end{aligned} \tag{72}$$

with

$$\eta = \frac{Q^2}{4\,M_d^2} \qquad q^2 = -Q^2\,. \tag{73}$$

The structure functions are defined by

$$F_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X) = \sum_{\mathcal{C}\in\{\mathcal{C}_\mathcal{V},\mathcal{C}_\mathcal{A}\}} \sum_{c',c\in\mathcal{C}} f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X;\,c',c)\,, \tag{74}$$

$$\widetilde{F}_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X) = 2 \sum_{c'\in\mathcal{C}_\mathcal{V},\,c\in\mathcal{C}_\mathcal{A}} f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X;\,c',c)\,. \tag{75}$$

Here the various current contributions $f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X;\,c',c)$ are given in terms of the quantities $\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c)$ defined below by

$$\begin{aligned}
&f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X;\,c',c) = c_{M'\mathrm{sig}_{M'}}\\
&\times\Big(\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM\mathrm{sig}_M}(c',c) + \mathrm{sig}_{M'}(-)^{M'}\mathcal{O}_{\alpha,\,I'-M'}^{(\prime)\,IM\mathrm{sig}_M}(c',c)\Big),
\end{aligned} \tag{76}$$

with

$$\begin{aligned}
&\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM\mathrm{sig}_M}(c',c) =\\
&c_{M\mathrm{sig}_M}\Big(\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) + \mathrm{sig}_M\,(-)^M \mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,I-M}(c',c)\Big).
\end{aligned} \tag{77}$$

The basic quantities are related to the t-matrix elements according to

$$\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) = \sum_{\lambda',\lambda} \delta_{\lambda\lambda'}^{(\prime)\,\alpha}\, \mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c)\,, \tag{78}$$

where the $\mathcal{U}$'s are quadratic Hermitian forms in the t-matrix elements

$$\begin{aligned}
&\mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c) =\\
&\frac{1}{6}\sum_{n',n,m',m}\Big(t_{n'\lambda'n}^{c'\,*}\,(\tau_{M'}^{[I']})_{n'm'}\, t_{m'\lambda m}^{c}\,(\tau_M^{[I]})_{mn} + (c'\leftrightarrow c)\Big).
\end{aligned} \tag{79}$$

Angular momentum conservation leads to the selection rule

$$\lambda' - \lambda = M' + M\,. \tag{80}$$

Note, that by definition $\mathcal{U}$ and thus the structure functions are symmetric under the interchange $(c \leftrightarrow c')$. Furthermore, one has the following symmetry properties:

$$\Big(\mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c)\Big)^* = (-)^{\delta_c^T+\delta_{c'}^T}\,\mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c)\,, \tag{81}$$

$$\mathcal{U}_{I'M'}^{\lambda\lambda'\,IM}(c',c) = (-)^{M+M'}\Big(\mathcal{U}_{I'-M'}^{\lambda'\lambda\,I-M}(c',c)\Big)^*\,, \tag{82}$$

$$\mathcal{U}_{I'-M'}^{-\lambda'-\lambda\,I-M}(c',c) = (-)^{\delta^{PT}(c',c)+I+I'}\Big(\mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c)\Big)^*, \tag{83}$$

$$\mathcal{U}_{I'M'}^{\lambda'\lambda\,IM}(c',c) = (-)^{I+M+I'+M'+\delta_c^T+\delta_{c'}^T}\,\mathcal{U}_{IM}^{\lambda'\lambda\,I'M'}(c',c), \tag{84}$$

where we have introduced

$$\delta^{PT}(c',c) = \delta_{c'}^{PT} + \delta_c^{PT}\,. \tag{85}$$

The first two can be combined to yield the symmetry

$$\begin{aligned}
&\mathcal{U}_{I'M'}^{-\lambda'-\lambda\,IM}(c',c) =\\
&\qquad(-)^{\delta^{PT}(c',c)+I+M+I'+M'}\,\mathcal{U}_{I'M'}^{\lambda\lambda'\,IM}(c',c)\,.
\end{aligned} \tag{86}$$

These symmetries are derived in the Appendix A, where we also give a closed expression for $\mathcal{U}_{I'M'}^{\lambda\lambda'\,IM}(c',c)$ in terms of the reduced multipole matrix elements. Furthermore, by a proper choice of the phases for the state vectors in order to have simple time reversal properties one can make all $\mathcal{U}_{I'M'}^{\lambda\lambda'\,IM}(c',c)$'s real or imaginary depending on whether $(-)^{\delta_c^T+\delta_{c'}^T} = \pm 1$, respectively, as also shown in the Appendix A. Then one finds corresponding simple symmetries for the $\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c)$

$$\Big(\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c)\Big)^* = (-)^{\delta_c^T+\delta_{c'}^T}\,\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c)\,, \tag{87}$$

$$\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) = (-)^{M+M'}\Big(\mathcal{O}_{\alpha,\,I'-M'}^{I-M}(c',c)\Big)^*\,, \tag{88}$$

$$\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) = \pm(-)^{\delta^{PT}(c',c)+I+I'}\,\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c)\,, \tag{89}$$

where the minus sign in (89) refers to the primed quantity $\mathcal{O}'^{\,IM}_{\alpha,\,I'M'}(c',c)$. For the interchange $(IM) \leftrightarrow (I'M')$ one has

$$\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) = (-)^{I+M+I'+M'+\delta_c^T+\delta_{c'}^T}\,\mathcal{O}_{\alpha,\,IM}^{(\prime)\,I'M'}(c',c). \tag{90}$$

The symmetry property of (89) leads to the interesting selection rule

$$\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c) = \frac{1}{2}\Big(1 \pm (-)^{\delta^{PT}(c',c)+I+I'}\Big)\,\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM}(c',c), \tag{91}$$

which means that

$$\begin{aligned}
&f_\alpha^{IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = 0 \quad \text{for } (-)^{\delta^{PT}(c',c)+I+I'} = -1,\\
&f_\alpha'^{\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = 0 \quad \text{for } (-)^{\delta^{PT}(c',c)+I+I'} = 1\,.
\end{aligned} \tag{92}$$

Another selection rule follows from (87) and (88) yielding

$$f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = c_{M'\mathrm{sig}_{M'}}\Big(1+\mathrm{sig}_M\mathrm{sig}_{M'}(-)^{\delta_c^T+\delta_{c'}^T}\Big)\,\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM\mathrm{sig}_M}(c',c). \quad (93)$$

Therefore, only for $\mathrm{sig}_M = \mathrm{sig}_{M'}(-)^{\delta_c^T+\delta_{c'}^T}$ one has a nonvanishing contribution. Combining these two selection rules and introducing as a shorthand

$$\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{(\prime)\,I,\,I'}(c',c) = \frac{1}{4}\Big(1+\mathrm{sig}_M\mathrm{sig}_{M'}(-)^{\delta_c^T+\delta_{c'}^T}\Big)\Big(1\pm(-)^{\delta^{PT}(c',c)+I+I'}\Big), \quad (94)$$

one obtains

$$f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = 2\,\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{(\prime)\,I,\,I'}(c',c)\,c_{M'\mathrm{sig}_{M'}}\,\mathcal{O}_{\alpha,\,I'M'}^{(\prime)\,IM\mathrm{sig}_M}(c',c)\,. \quad (95)$$

The remaining nonvanishing functions are listed in table 1. In detail, one finds for them (note that by definition $M,\,M'\geq 0$)

$$f_L^{IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = 2\,\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{I,\,I'}(c',c)\,c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}}\,\delta_{M'M} \times\Big(\delta_{M0}+\mathrm{sig}_M(-)^M\Big)\,\mathcal{U}_{I'M}^{00\,I-M}(c',c)\,, \quad (96)$$

$$f_T^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = 4\,\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{(\prime)\,I,\,I'}(c',c)\,c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}}\,\delta_{M'M} \times\Big(\delta_{M0}+\mathrm{sig}_M(-)^M\Big)\,\mathcal{U}_{I'M}^{11\,I-M}(c',c)\,, \quad (97)$$

$$\begin{aligned} f_{LT}^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = &-4\,\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{(\prime)\,I,\,I'}(c',c)\,c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}}\\ &\times\Big(\mathrm{sig}_M(-)^{M+\delta_c^T+\delta_{c'}^T}\delta_{M',\,M+1}\,\mathcal{U}_{I'-M-1}^{01\,IM}(c',c)\\ &+((-)^{\delta_c^T+\delta_{c'}^T}\delta_{M',\,1-M}-\mathrm{sig}_M(-)^M\delta_{M',\,M-1})\\ &\times\mathcal{U}_{I'M-1}^{01\,I-M}(c',c)\Big)\,, \end{aligned} \quad (98)$$

$$\begin{aligned} f_{TT}^{IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = &\,2\,\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{I,\,I'}(c',c)\,c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}}\\ &\times\Big(\mathrm{sig}_M(-)^{M+\delta_c^T+\delta_{c'}^T}\delta_{M',\,M+2}\,\mathcal{U}_{I'-M-2}^{-11\,IM}(c',c)\\ &+((-)^{\delta_c^T+\delta_{c'}^T}\delta_{M',\,2-M}+\mathrm{sig}_M(-)^M\delta_{M',\,M-2})\\ &\times\mathcal{U}_{I'M-2}^{-11\,I-M}(c',c)\Big)\,. \end{aligned} \quad (99)$$

The symmetry property in (90) leads to a simple relation for the interchange $(IM\mathrm{sig}_M \leftrightarrow I'M'\mathrm{sig}_{M'})$

$$f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c) = (-)^{I+M+I'+M'+\delta_c^T+\delta_{c'}^T}\,f_\alpha^{(\prime)\,I'M'\mathrm{sig}_{M'}}(IM\mathrm{sig}_M;c',c), \quad (100)$$

which relates the f-functions for a given target and recoil polarization to the corresponding ones, where the target and recoil polarizations have been interchanged. Thus this symmetry reduces the number of independent structure functions considerably and gives an additional selection rule for $(IM\mathrm{sig}_M) = (I'M'\mathrm{sig}_{M'})$

$$f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(IM\mathrm{sig}_M;\,c',c) = 0\,, \quad (101)$$

for $\delta_c^T+\delta_{c'}^T = 1$. Another symmetry exists for the structure functions with $M>0$ and $M'>0$ for the simultaneous sign change $\mathrm{sig}_M\to-\mathrm{sig}_M$ and $\mathrm{sig}_{M'}\to-\mathrm{sig}_{M'}$. First we note from (60) the property

$$c_{M\,-\mathrm{sig}_M} = i\,\mathrm{sig}_M\,c_{M\mathrm{sig}_M}\,, \quad (102)$$

from which follows for the above transformation:

$$c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}} \to -\mathrm{sig}_M\,\mathrm{sig}_{M'}\,c_{M\mathrm{sig}_M}\,c_{M'\mathrm{sig}_{M'}}\,. \quad (103)$$

Secondly, the invariance of $\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{I,\,I'}(c',c)$ is evident. Finally, from the formal expressions in (96) through (99) one notes that for $M>0$ and $M'>0$ $f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c)$ is proportional to sig_M for $\alpha\in\{L,T,LT\}$, whereas for $\alpha=TT$ it is independent because in this case only $M=M'=1$ gives a nonvanishing contribution to (99) according to table 2. Thus with (103) and the equivalence $\mathrm{sig}_M\,\mathrm{sig}_{M'}\equiv(-)^{\delta_{c'}^T+\delta_c^T}$ implied by $\delta_{\mathrm{sig}_M\mathrm{sig}_{M'}}^{I,\,I'}(c',c)$, one finds

$$f_\alpha^{(\prime)\,IM\,-\mathrm{sig}_M}(I'M'-\mathrm{sig}_{M'};\,c',c) = (-)^{\delta_{c'}^T+\delta_c^T}\,f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c)\,, \quad (104)$$

for $\alpha\in\{L,T,LT\}$ and

$$f_{TT}^{(\prime)\,IM\,-\mathrm{sig}_M}(I'M'-\mathrm{sig}_{M'};\,c',c) = -(-)^{\delta_{c'}^T+\delta_c^T}\,f_{TT}^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c)\,. \quad (105)$$

Table 1. Listing of the $\mathrm{sig}_{M'}$- and $(I+I')$-values for the nonvanishing $f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c)$ for various current contributions.

				$I'+I$	
c'-c	$(-)^{\delta_{c'}^T+\delta_c^T}$	$(-)^{\delta(c',c)}$	$\mathrm{sig}_{M'}$	Unprimed	Primed
pctc-pctc	1	1	sig_M	even	odd
pnctc-pctc	1	−1	sig_M	odd	even
pctnc-pctc	−1	−1	$-\mathrm{sig}_M$	odd	even
pnctnc-pctc	−1	1	$-\mathrm{sig}_M$	even	odd
pctnc-pnctc	−1	1	$-\mathrm{sig}_M$	even	odd
pnctnc-pnctc	−1	−1	$-\mathrm{sig}_M$	odd	even
pnctnc-pctnc	1	−1	sig_M	odd	even

Table 2. Listing of the (M,M')-values for the $f_\alpha^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'};\,c',c)$.

α	(M,M')
L, T	(0,0), (1,1), (2,2)
LT	(0,1), (1,0), (1,2), (2,1)
TT	(0,2), (1,1), (2,0)

Explicit expressions for the nonvanishing f-functions for the various current contributions for the case of recoil polarization without target polarization are listed in the Appendix B. The ones for target polarization without analysis of the recoil polarization can be obtained from these by using the symmetry in (100).

Besides the dominant P- and T-invariance obeying contribution $c = \gamma_{pc,\,tc}$, we now will restrict ourselves to the first-order contributions with respect to the weak coupling constant and to leading order T-violation. In other words, of all the contributions of $\mathcal{C}_\mathcal{V}$ and $\mathcal{C}_\mathcal{A}$ the following zero and first-order contributions remain:

$$\begin{aligned}
\mathcal{C}^{(0)}_\mathcal{V} &= \Big\{\gamma_{pc,\,tc}\Big\}\,,\\
\mathcal{C}^{(0)}_\mathcal{A} &= \emptyset\,,\\
\mathcal{C}^{(1)}_\mathcal{V} &= \Big\{\gamma_{pnc,\,tc},\,\gamma_{pc,\,tnc},\,Z^\mathcal{V}_{v,\,pc,\,tc},\,Z^\mathcal{V}_{a,\,pc,\,tc}\Big\}\,,\\
\mathcal{C}^{(1)}_\mathcal{A} &= \Big\{Z^\mathcal{A}_{v,\,pc,\,tc},\,Z^\mathcal{A}_{a,\,pc,\,tc}\Big\}\,,
\end{aligned} \tag{106}$$

or with respect to the other classification of (45) through (48):

$$\begin{aligned}
\mathcal{C}^{(0)}_{pc,\,tc} &= \Big\{\gamma_{pc,\,tc}\Big\}\,,\\
\mathcal{C}^{(1)}_{pc,\,tc} &= \Big\{Z^\mathcal{V}_{v,\,pc,\,tc},\,Z^\mathcal{A}_{v,\,pc,\,tc}\Big\}\,,\\
\mathcal{C}^{(1)}_{pnc,\,tc} &= \Big\{\gamma_{pnc,\,tc},\,Z^\mathcal{V}_{a,\,pc,\,tc},\,Z^\mathcal{A}_{a,\,pc,\,tc}\Big\}\,,\\
\mathcal{C}^{(1)}_{pc,\,tnc} &= \Big\{\gamma_{pc,\,tnc}\Big\}\,,\\
\mathcal{C}^{(1)}_{pnc,\,tnc} &= \emptyset\,.
\end{aligned} \tag{107}$$

This means, one has to make the following substitutions in the general expression for the structure functions in (74) and (75)

$$\sum_{\mathcal{C}=(\mathcal{C}_\mathcal{V},\,\mathcal{C}_\mathcal{A})}\ \sum_{c',\,c\in\mathcal{C}} \to \sum_{c'=c=\gamma_{pc,\,tc}} + 2\sum_{c'=\gamma_{pc,\,tc},\,c\in\mathcal{C}^{(1)}_\mathcal{V}}\,, \tag{108}$$

$$\sum_{c'\in\mathcal{C}_\mathcal{V},\,c\in\mathcal{C}_\mathcal{A}} \to \sum_{c'=\gamma_{pc,\,tc},\,c\in\{Z^\mathcal{A}_{v,\,pc,\,tc},Z^\mathcal{A}_{a,\,pc,\,tc}\}}\,, \tag{109}$$

and obtains for the structure functions one diagonal P- and T-conserved contribution ($c = \gamma_{pc,\,tc}$) to $F^{(\prime)\,IM\mathrm{sig}_M}_\alpha(X)$ and three nondiagonal P- or T-violating ones, namely two P-violating contributions from the hadronic P-violation ($c = \gamma_{pnc,\,tc}$) and from the hadronic axial current coupled to the lepton vector current ($c = Z^\mathcal{V}_{a,\,pc,\,tc}$), and one hadronic T-violating contribution ($\gamma_{pc,\,tnc}$). Here and in the following, "X" stands always for an observable "$I'M'\mathrm{sig}_{M'}$". The corresponding structure functions are determined either by the P- and T-conserved f-functions or by the P- or T-violating ones. In view of the selection rules for $\delta^{I,\,I'}_{\mathrm{sig}_M\mathrm{sig}_{M'}}(c',c)$ contained in (94), one finds in detail for the P- and T-conserving structure functions for which $\mathrm{sig}_M = \mathrm{sig}_{M'}$ applies,

$$\begin{aligned}
&F^{IM\mathrm{sig}_M}_\alpha(X) = f^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},\gamma_{pc,tc})\\
&+2f^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},Z^\mathcal{V}_{v,pc,tc}), \qquad \text{for } I+I' \text{ even},
\end{aligned}\tag{110}$$

$$\begin{aligned}
&F'^{IM\mathrm{sig}_M}_\alpha(X) = f'^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},\gamma_{pc,tc})\\
&+2f'^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},Z^\mathcal{V}_{v,pc,tc}), \qquad \text{for } I+I' \text{ odd}.
\end{aligned}\tag{111}$$

Taking into account the proportionality of the neutral vector current to the e.m. current

$$J^{Z_v}_\mu = g^d_v\,J^\gamma_\mu\,, \tag{112}$$

where $g^d_v = -2\sin^2\theta_\mathrm{W}$, then one obtains

$$\begin{aligned}
F^{IM\mathrm{sig}_M}_\alpha(X) &= (1+2\,g^d_v\,\widetilde{G}_v)\,f^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},\gamma_{pc,\,tc})\,,\\
&\qquad \text{for } I+I' \text{ even},
\end{aligned}\tag{113}$$

$$\begin{aligned}
F'^{IM\mathrm{sig}_M}_\alpha(X) &= (1+2g^d_v\widetilde{G}_v)f'^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},\gamma_{pc,tc}),\\
&\qquad \text{for } I+I' \text{ odd},
\end{aligned}\tag{114}$$

which means a simple renormalization by a factor almost unity. Furthermore, for the P-violating structure functions, for which also $\mathrm{sig}_M = \mathrm{sig}_{M'}$ applies, one has

$$\begin{aligned}
&F^{IM\mathrm{sig}_M}_\alpha(X) = 2f^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},\gamma_{pnc,tc})\\
&+2f^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},Z^\mathcal{V}_{a,pc,tc}) \quad \text{for } I+I' \text{ odd},
\end{aligned}\tag{115}$$

$$\begin{aligned}
&F'^{IM\mathrm{sig}_M}_\alpha(X) = 2f'^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},\gamma_{pnc,tc})\\
&+2f'^{IM\mathrm{sig}_M}_\alpha(X;\gamma_{pc,tc},Z^\mathcal{V}_{a,pc,tc}) \quad \text{for } I+I' \text{ even}.
\end{aligned}\tag{116}$$

Finally, for the T-violating structure functions, for which $\mathrm{sig}_M = -\mathrm{sig}_{M'}$ applies, one finds

$$\begin{aligned}
F^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},\gamma_{pc,\,tnc})\\
&\qquad \text{for } I+I' \text{ odd},
\end{aligned}\tag{117}$$

$$\begin{aligned}
F'^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f'^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},\gamma_{pc,\,tnc})\\
&\qquad \text{for } I+I' \text{ even}.
\end{aligned}\tag{118}$$

To $\widetilde{F}^{(\prime)\,IM\mathrm{sig}_M}_\alpha(X)$ one has two nondiagonal P-violating contributions from the neutral hadron current, containing vector and axial pieces, coupled to the axial lepton current, *i.e.*,

$$\begin{aligned}
\widetilde{F}^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},Z^\mathcal{A}_{v,\,pc,\,tc})\\
&\qquad \text{for } I+I' \text{ even},
\end{aligned}\tag{119}$$

$$\begin{aligned}
\widetilde{F}^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},Z^\mathcal{A}_{a,\,pc,\,tc})\\
&\qquad \text{for } I+I' \text{ odd},
\end{aligned}\tag{120}$$

$$\begin{aligned}
\widetilde{F}'^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f'^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},Z^\mathcal{A}_{v,\,pc,\,tc})\\
&\qquad \text{for } I+I' \text{ odd},
\end{aligned}\tag{121}$$

$$\begin{aligned}
\widetilde{F}'^{IM\mathrm{sig}_M}_\alpha(X) &= 2\,f'^{IM\mathrm{sig}_M}_\alpha(X;\,\gamma_{pc,\,tc},Z^\mathcal{A}_{a,\,pc,\,tc})\\
&\qquad \text{for } I+I' \text{ even},
\end{aligned}\tag{122}$$

where again $\mathrm{sig}_M = \mathrm{sig}_{M'}$ applies. Explicit expressions for the nonvanishing structure functions are listed in Appendix C.

It is useful to introduce scalar, vector and tensor target asymmetries $A^I_d(X)$ ($I = 0,1,2$) and corresponding beam-target asymmetries $A^I_{ed}(X)$ which can be separated by a proper variation of the electron polarization parameter h and the target polarization parameters P^d_I. They are defined by

$$\begin{aligned}
\mathcal{O}_X\,\frac{d\sigma^{\gamma+Z}}{d\Omega^\mathrm{lab}_{k_2}} &= \sigma_\mathrm{Mott}\,S_0\,\Big[A^0_d(X)+P^d_1\,A^1_d(X)+P^d_2\,A^2_d(X)\\
&+h\,\Big(A^0_{ed}(X)+P^d_1\,A^1_{ed}(X)+P^d_2\,A^2_{ed}(X)\Big)\Big]\,,
\end{aligned}\tag{123}$$

where

$$S_0 = v_L\, F_L^{00+}(1) + v_T\, F_T^{00+}(1)\,. \tag{124}$$

Note that $A_d^0(X=1) = A_d^0(00+) = 1$. Comparison with (70) yields the following expressions:

$$A^I_{d/ed}(X) = \sum_{M=0}^{I} d^I_{M0}(\theta_d) \times \sum_{\mathrm{sig}_M} \cos\Big(M\phi_d + \frac{\pi}{4}(1-\mathrm{sig}_M 1)\Big)\, A_d^{IM\mathrm{sig}_M}(X)\,, \tag{125}$$

or in detail

$$A^0_{d/ed}(X) = A^{00+}_{d/ed}(X)\,, \tag{126}$$

$$A^1_{d/ed}(X) = \cos\theta_d\, A^{10+}_{d/ed}(X) - \frac{\sin\theta_d}{\sqrt{2}}\Big(\cos\phi_d\, A^{11+}_{d/ed}(X) - \sin\phi_d\, A^{11-}_{d/ed}(X)\Big)\,, \tag{127}$$

$$A^2_{d/ed}(X) = \frac{1}{2}\,(3\,\cos\theta_d - 1)\, A^{20+}_{d/ed}(X) - \sqrt{\frac{3}{2}}\,\sin\theta_d\,\cos\theta_d\,\Big(\cos\phi_d\, A^{21+}_{d/ed}(X) - \sin\phi_d\, A^{21-}_{d/ed}(X)\Big) + \frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^2\theta_d\,\Big(\cos 2\phi_d\, A^{22+}_{d/ed}(X) - \sin 2\phi_d\, A^{22-}_{d/ed}(X)\Big)\,, \tag{128}$$

where we have separated explicitly the dependence on the angles of the deuteron orientation axis by introducing

$$A_d^{IM\mathrm{sig}_M}(X) = \frac{1}{S_0} \times \sum_{\alpha\in\{L,T,LT,TT\}} \Big[v_\alpha F_\alpha^{IM\mathrm{sig}_M}(X) + v'_\alpha \widetilde{F}_\alpha'^{\,IM\mathrm{sig}_M}(X)\Big], \tag{129}$$

$$A_{ed}^{IM\mathrm{sig}_M}(X) = \frac{1}{S_0} \times \sum_{\alpha\in\{L,T,LT,TT\}} \Big[v_\alpha \widetilde{F}_\alpha^{IM\mathrm{sig}_M}(X) + v'_\alpha F_\alpha'^{\,IM\mathrm{sig}_M}(X)\Big]. \tag{130}$$

The latter asymmetries $A_{e/ed}^{IM\mathrm{sig}_M}(X)$ can be separated by a proper choice of the orientation angles θ_d and ϕ_d. We list explicit expressions of all asymmetries for $\mathrm{sig}_M = +$ in the Appendix D except for those which can be obtained from the symmetry in (100), *i.e.*, from

$$A_{d/ed}^{(\prime)\,IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'}) = (-)^{I+M+I'+M'+\delta_t}\, A_{d/ed}^{(\prime)\,I'M'\mathrm{sig}_{M'}}(IM\mathrm{sig}_M)\,, \tag{131}$$

where $\delta_t = 1$ for the T-violating contributions, and $\delta_t = 0$ else. The ones for $\mathrm{sig}_M = -$ can be obtained with the help of the relations in (104) and (105) yielding

$$A_d^{IM-\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'}) = \frac{1}{S_0}\sum_{\alpha\in\{L,T,LT,TT\}} (-)^{\delta_{\alpha,TT}+\delta_t}\Big[v_\alpha F_\alpha^{IM\mathrm{sig}_M}(I'M'-\mathrm{sig}_{M'}) + v'_\alpha \widetilde{F}_\alpha'^{\,IM\mathrm{sig}_M}(I'M'{-}\mathrm{sig}_{M'})\Big] = (-)^{\delta_t}\Big(A_d^{IM\mathrm{sig}_M}(I'M'{-}\mathrm{sig}_{M'}) - \frac{2}{S_0}v_{TT}F_{TT}^{IM\mathrm{sig}_M}(I'M'{-}\mathrm{sig}_{M'})\Big)\,, \tag{132}$$

$$A_{ed}^{IM-\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'}) = (-)^{\delta_t}\Big(A_{ed}^{IM\mathrm{sig}_M}(I'M'-\mathrm{sig}_{M'}) - \frac{2}{S_0}\,v_{TT}\,\widetilde{F}_{TT}^{IM\mathrm{sig}_M}(I'M'-\mathrm{sig}_{M'})\Big)\,. \tag{133}$$

With respect to the explicit expressions of Appendix D one should keep in mind the relation (112) of the neutral hadronic vector current of the deuteron $J_\mu^{Z_v}$, which means that the P- and T-conserving form factors of $J_\mu^{Z_v}$ are proportional to the corresponding e.m. form factors with g_v^d as proportionality constant. In particular, this means for the neutral current form factors appearing in the P-violating asymmetries of Appendix D according to (32) and (33)

$$C_L^{Z_v^{\mathcal{V}/\mathcal{A}}} = g_v^d\,\widetilde{G}_{v/a}\,C_L^\gamma \quad\text{and}\quad M_1^{Z_v^{\mathcal{V}/\mathcal{A}}} = g_v^d\,\widetilde{G}_{v/a}\,M_1^\gamma\,, \tag{134}$$

where $\widetilde{G}_{v/a} = \sqrt{2}\,g^e_{v/a}\,\widetilde{G}_\mathrm{F}\,q_\mu^2\,e^{-2}$ (see (13)). Furthermore, in the P- and T-conserving asymmetries the e.m. form factors become renormalized by a factor very close to unity according to (113) and (114). Finally, the P-violating $E1$-multipole contributions $E_1^{Z_a^{\mathcal{V}/\mathcal{A}}}$ of the axial part of the hadronic neutral current $J_\mu^{Z_a}$ are related to the deuteron axial form factor

$$F_{E1}^A = \frac{\sqrt{E'_d E_d}}{M_d}\,\langle 1\|\mathcal{E}_1(J_\mu^{Z_a})\|1\rangle \tag{135}$$

by

$$E_1^{Z_a^{\mathcal{V}/\mathcal{A}}} = \widetilde{G}_{v/a}\,F_{E1}^A\,. \tag{136}$$

At the end of this section, we will furthermore introduce the usual invariant multipole form factors and structure functions depending on Q^2 alone by

$$G_C = \sqrt{\frac{4\,\pi}{3}}\,\frac{\beta}{1+\eta}\,C_0\,, \tag{137}$$

$$G_Q = \sqrt{\frac{3\,\pi}{2}}\,\frac{\beta}{\eta(1+\eta)}\,C_2\,, \tag{138}$$

$$G_{(E/M)L} = \sqrt{\frac{\pi}{\eta\,(1+\eta)}}\,(E/M)_L\,. \tag{139}$$

and

$$G_L^{IM\mathrm{sig}_M}(X) = \frac{\beta^2}{(1+\eta)^2}\, F_L^{IM\mathrm{sig}_M}(X)\,, \qquad (140)$$

$$G_T^{(\prime)\,IM\mathrm{sig}_M}(X) = \frac{1}{2\,\eta\,(1+\eta)}\, F_T^{(\prime)\,IM\mathrm{sig}_M}(X)\,, \qquad (141)$$

$$G_{LT}^{(\prime)\,IM\mathrm{sig}_M}(X) = \frac{\beta}{(1+\eta)\sqrt{2\eta(1+\eta)}} F_{LT}^{(\prime)IM\mathrm{sig}_M}(X), \qquad (142)$$

$$G_{TT}^{IM\mathrm{sig}_M}(X) = \frac{1}{2\,\eta\,(1+\eta)}\, F_{TT}^{IM\mathrm{sig}_M}(X)\,, \qquad (143)$$

and corresponding relations for the $\widetilde{G}_\alpha^{(\prime)\,IM\mathrm{sig}_M}(X)$ structure functions. In terms of these invariant structure functions the asymmetries in (130) and (130) become

$$A_d^{IM\mathrm{sig}_M}(X) = \frac{1}{S_0} \times \sum_{\alpha\in\{L,T,LT,TT\}} \Big[\widetilde{v}_\alpha G_\alpha^{IM\mathrm{sig}_M}(X) + \widetilde{v}'_\alpha \widetilde{G}_\alpha^{\prime IM\mathrm{sig}_M}(X)\Big], \qquad (144)$$

$$A_{ed}^{IM\mathrm{sig}_M}(X) = \frac{1}{S_0} \times \sum_{\alpha\in\{L,T,LT,TT\}} \Big[\widetilde{v}_\alpha \widetilde{G}_\alpha^{IM\mathrm{sig}_M}(X) + \widetilde{v}'_\alpha G_\alpha^{\prime IM\mathrm{sig}_M}(X)\Big], \qquad (145)$$

with

$$S_0 = {G_C}^2 + \frac{8}{9}\,\eta^2\,{G_Q}^2 + \frac{2}{3}\,\eta\,\Big(1 + 2\,(1+\eta)\tan^2\frac{\theta}{2}\Big)\,{G_M}^2\,, \qquad (146)$$

and

$$\begin{aligned}
\widetilde{v}_L &= \tfrac{(1+\eta)^2}{\beta^2}\, v_L\,,\\
\widetilde{v}_T^{(\prime)} &= 2\,\eta\,(1+\eta)\, v_T^{(\prime)}\,,\\
\widetilde{v}_{LT}^{(\prime)} &= \tfrac{1}{\beta}\,(1+\eta)\sqrt{2\,\eta\,(1+\eta)}\, v_{LT}^{(\prime)}\,,\\
\widetilde{v}_{TT} &= 2\,\eta\,(1+\eta)\, v_{TT}\,,
\end{aligned} \qquad (147)$$

or in explicit form

$$\begin{aligned}
\widetilde{v}_L &= 1\,,\\
\widetilde{v}_T &= \eta\,\Big(1 + 2\,(1+\eta)\,\tan^2\tfrac{\theta_e^{\mathrm{lab}}}{2}\Big)\,,\\
\widetilde{v}_{LT} &= \sec\tfrac{\theta_e^{\mathrm{lab}}}{2}\,\sqrt{\eta\,(1+\eta\,\sin^2\tfrac{\theta_e^{\mathrm{lab}}}{2})}\,,\\
\widetilde{v}_{TT} &= -\eta\,,\\
\widetilde{v}'_{LT} &= \tan\tfrac{\theta_e^{\mathrm{lab}}}{2}\,\sqrt{\eta\,(1+\eta)}\,,\\
\widetilde{v}'_T &= 2\,\sec\tfrac{\theta_e^{\mathrm{lab}}}{2}\,\tan\tfrac{\theta_e^{\mathrm{lab}}}{2}\,\eta\,\sqrt{(1+\eta)(1+\eta\,\sin^2\tfrac{\theta_e^{\mathrm{lab}}}{2})}\,.
\end{aligned} \qquad (148)$$

Detailed expressions of the resulting asymmetries are listed in Appendix E. Similarly to what has been said with respect to Appendix D above, we would like to remind the reader that relations analogous to (134) exist also for the P- and T-conserving neutral invariant form factors, namely

$$G_{C/Q}^{Z_v^{\mathcal{V}/\mathcal{A}}} = g_v^d\,\widetilde{G}_{v/a}\,G_{C/Q} \quad \text{and} \quad G_M^{Z_v^{\mathcal{V}/\mathcal{A}}} = g_v^d\,\widetilde{G}_{v/a}\,G_M\,, \qquad (149)$$

and that one has the relation of the P-violating invariant form factors $G_{E1}^{Z_a^{\mathcal{V}/\mathcal{A}}}$ to the deuteron invariant axial form factor

$$G_{E1}^A = \sqrt{\frac{\pi}{\eta\,(1+\eta)}}\,F_{E1}^A\,, \qquad (150)$$

which reads

$$G_{E1}^{Z_a^{\mathcal{V}/\mathcal{A}}} = \widetilde{G}_{v/a}\,G_{E1}^A\,. \qquad (151)$$

5 Discussion and summary

A schematic survey of the nonvanishing asymmetries is given in tables 3 through 5 where we have not listed those which are related to the listed ones by the above mentioned symmetries. The simplest asymmetries to measure are the scalar asymmetries in table 3 involving the determination of the deuteron recoil polarization for an unpolarized deuteron target without or with longitudinal electron polarization, or for the equivalent situation using an oriented deuteron target but not measuring the recoil polarization. We will discuss these scalar asymmetries in some detail. The vector and tensor asymmetries do not provide additional information but they may be used for independent checks.

5.1 P- and T-conserving contributions

For the P- and T-conserving currents one finds as scalar asymmetries only tensor recoil polarization components, if the electrons are unpolarized as is well known, and our results for them agree with the ones given in the literature,

$$S_0\,A_d^{00\,+}(20\,+) = S_0\,T_{20} = -\frac{\eta}{3\,\sqrt{2}} \times\Big(8(G_C+\frac{\eta}{3}G_Q)G_Q+(1+2\,(1+\eta)\tan^2\frac{\theta}{2}){G_M}^2\Big), \qquad (152)$$

$$S_0\,A_d^{00\,+}(21\,+) = S_0\,T_{21} = \frac{4}{\sqrt{3}}\,\sec\frac{\theta}{2}\,\eta\,\sqrt{\eta\,\Big(1+\eta\sin^2\frac{\theta}{2}\Big)}\,G_M\,G_Q, \qquad (153)$$

$$S_0\,A_d^{00\,+}(22\,+) = S_0\,T_{22} = -\frac{\eta}{\sqrt{3}}\,{G_M}^2\,. \qquad (154)$$

In particular, with respect to the expressions given in eq. (5.11) of [2], using Schildknecht's notation, one finds

$$s'^{\,11} = P_{zz} = \frac{\sqrt{2}}{3}\,\mathcal{O}_{20+} = \frac{\sqrt{2}}{3}\,A_d^{00+}(20+)\,, \qquad (155)$$

$$s'^{\,22} = P_{xx} = \frac{1}{2\,\sqrt{3}}\,\mathcal{O}_{22+} - \frac{1}{3\,\sqrt{2}}\,\mathcal{O}_{20+} = \frac{1}{2\,\sqrt{3}}\,A_d^{00+}(22+) - \frac{1}{3\,\sqrt{2}}\,A_d^{00+}(20+), \qquad (156)$$

$$s'^{\,12} = P_{zx} = -\frac{1}{2\,\sqrt{3}}\,\mathcal{O}_{21+} = -\frac{1}{2\,\sqrt{3}}\,A_d^{00+}(21+)\,. \qquad (157)$$

Table 3. Schematic survey of nonvanishing scalar asymmetries $A^{00+}_{d/ed}(I'M'\mathrm{sig}_{M'})$.

Type	Current	00+	10+	11+	11−	20+	21+	21−	22+	22−
	PT-conserving	$\surd$				$\surd$	$\surd$		$\surd$	
$A^{00+}_{d}(I'M'\mathrm{sig}_{M'})$	P-violating	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$			
	T-violating				$\surd$					
	PT-conserving		$\surd$	$\surd$						
$A^{00+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$		$\surd$	
	T-violating							$\surd$		

Table 4. Schematic survey of nonvanishing vector asymmetries $A^{1M+}_{d/ed}(I'M'\mathrm{sig}_{M'})$ for $I' \geq 1$.

Type	Current	10+	11+	11−	20+	21+	21−	22+	22−
	PT-conserving	$\surd$	$\surd$						
$A^{10+}_{d}(I'M'\mathrm{sig}_{M'})$	P-violating		$\surd$		$\surd$	$\surd$			
	T-violating						$\surd$		
	PT-conserving		$\surd$						
$A^{11+}_{d}(I'M'\mathrm{sig}_{M'})$	P-violating		$\surd$			$\surd$		$\surd$	
	T-violating						$\surd$		$\surd$
	PT-conserving				$\surd$	$\surd$			
$A^{10+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating	$\surd$	$\surd$		$\surd$	$\surd$			
	T-violating			$\surd$					
	PT-conserving					$\surd$		$\surd$	
$A^{11+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating		$\surd$			$\surd$		$\surd$	
	T-violating			$\surd$					

The tensor component T_{20} is used to separate the charge from the quadrupole form factor, while T_{21} allows to determine the relative phase between the magnetic and quadrupole form factor. The component T_{22} does not provide new information, it could only be taken as an independent check of the structure function $B(Q^2)$ because one would not need to perform a Rosenbluth separation.

With additional longitudinal electron polarization one finds as scalar asymmetries for the leading order P- and T-conserving currents two vector recoil polarization components, again in agreement with the ones given in eq. (5.16) of [2], taking into account the relations

$$\frac{{}^2c_{11}}{{}^1a} = P_z = \sqrt{\frac{2}{3}}\,\mathcal{O}_{10+} = \sqrt{\frac{2}{3\,S_0}}\,A^{00+}_{ed}(10+)$$
$$= \frac{2}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_M^2, \quad (158)$$
$$\frac{{}^2c_{21}}{{}^1a} = P_x = -\frac{1}{\sqrt{3}}\,\mathcal{O}_{11+} = -\frac{1}{\sqrt{3}}\,A^{00+}_{ed}(11+)$$
$$= -\frac{4}{3\,S_0}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_C+\frac{\eta}{3}\,G_Q\right)G_M\,. \quad (159)$$

The first one, P_z, is proportional to G_M^2, whereas the component perpendicular to the momentum transfer but in the scattering plane, P_x, contains interference of G_M with G_C and G_Q. The vector and tensor asymmetries listed in the Appendix E do not contain additional information but they could be used for consistency checks.

5.2 Parity-violating contributions

Parity violation gives a small contribution to the unpolarized cross-section from the $E1$ contribution $G_{E1}^{Z_a^A} = \widetilde{G}_a\,G^A_{E1}$ to the hadronic neutral axial current

$$S_0\,A^{00\,+}_d(00\,+) = \frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,\widetilde{G}_a\,G^A_{E1}\,G_M\,, \quad (160)$$

and also to some recoil tensor polarization components (see Appendix E) which, however, will be very difficult to disentangle from the leading-order contribution. One has to look for observables for which the leading-order contribution vanishes. According to table 3, the vector polarization components provide such observables. The axial form factor $G_{E1}^{Z_a^V} = \widetilde{G}_v\,G^A_{E1}$ of the hadronic neutral axial current as well as parity violation in the hadronic structure, manifest in a nonvanishing axial form factor G^γ_{E1}, induce vector polarization components in the scattering plane, P_x

Table 5. Schematic survey of nonvanishing tensor asymmetries $A^{2M+}_{d/ed}(I'M'\mathrm{sig}_{M'})$ for $I' \geq 2$.

Type	Current	20+	21+	21–	22+	22–
$A^{20+}_{d}(I'M'\mathrm{sig}_{M'})$	PT-conserving	√	√		√	
	P-violating	√	√			
$A^{21+}_{d}(I'M'\mathrm{sig}_{M'})$	PT-conserving		√		√	
	P-violating		√		√	
$A^{22+}_{d}(I'M'\mathrm{sig}_{M'})$	PT-conserving				√	
$A^{20+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating	√	√		√	
	T-violating			√		
$A^{21+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating		√		√	
	T-violating			√		√
$A^{22+}_{ed}(I'M'\mathrm{sig}_{M'})$	P-violating				√	

and P_z [6]. They are given by

$$\begin{aligned} S_0\, P_z = &\sqrt{\frac{2}{3}}\, S_0\, A_d^{00\,+}(10\,+) \\ = &\frac{2}{3}\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right)\left(G^{\gamma}_{E1}+\widetilde{G}_v\,G^{A}_{E1}\right)G_M \\ &+\frac{4}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \\ &\times\sqrt{(1+\eta)(1+\eta\,\sin^2\frac{\theta}{2})}\,g^d_v\,\widetilde{G}_a\,G^2_M\,, \end{aligned} \tag{161}$$

$$\begin{aligned} S_0\, P_x = &-\frac{1}{\sqrt{3}}\,S_0\,A_d^{00\,+}(11\,+) \\ = &-\frac{4}{3}\,\sec\frac{\theta}{2}\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)} \\ &\times\left(G^{\gamma}_{E1}+\widetilde{G}_v\,G^{A}_{E1}\right)\left(G_C+\frac{\eta}{3}\,G_Q\right) \\ &-\frac{8}{3}\tan\frac{\theta}{2}\sqrt{\eta(1+\eta)}\;g^d_v\widetilde{G}_a\left(G_C+\frac{\eta}{3}G_Q\right)G_M. \end{aligned} \tag{162}$$

Obviously, these observables allow one to determine only the combination of the axial form factors $G^{\gamma}_{E1}+\widetilde{G}_v\,G^{A}_{E1}$. However, one has to keep in mind that contributions proportional to $\widetilde{G}_v$ are suppressed by $(4\,\sin^2\theta_{\mathrm{W}}-1)$ compared to those proportional to $\widetilde{G}_a$.

The same combination of the axial form factors G^{γ}_{E1} and G^{A}_{E1} leads also to a nonvanishing asymmetry of the differential cross-section with respect to longitudinally polarized electrons without deuteron polarization [5,6] according to

$$\begin{aligned} S_0\,A_{ed}^{00\,+}(00\,+) = &\,2\,g^d_v\,\widetilde{G}_a\,S_0+\frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \\ &\times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\left(G^{\gamma}_{E1}+\widetilde{G}_v\,G^{A}_{E1}\right)G_M\,. \end{aligned} \tag{163}$$

With respect to the neutral hadron current contributions to the asymmetries in (161), (162), and (163), these expressions agree with those of [6] if one makes the following identifications:

$$\begin{aligned} &G_0\equiv G_C,\quad G_2\equiv\frac{2\sqrt{2}}{3}\,\eta\,G_Q,\quad G_1\equiv G_M, \\ &F_A\equiv\sqrt{\frac{\eta}{1+\eta}}\,G^{A}_{E1},\quad\text{and}\quad g^n_V\equiv 2\,g^d_v\,. \end{aligned} \tag{164}$$

Another contribution from P-violation via the larger form factor $G^{Z^{\mathcal{A}}_a}_{E1}=\widetilde{G}_a\,G^{A}_{E1}$ to observables, depending on the electron polarization, appears for the recoil vector polarization P_z

$$\begin{aligned} &S_0\,A_{ed}^{00\,+}(10\,+) = \\ &\sqrt{\frac{2}{3}}\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right)\widetilde{G}_a\,G^{A}_{E1}\,G_M\,, \end{aligned} \tag{165}$$

which, in principle, would allow one to determine separately the neutral current axial form factor G^{A}_{E1}. However, like $A_d^{00\,+}(00\,+)$ this observable will be buried by the leading order of (158). This is a general feature as a closer inspection of Appendix E shows, whenever $G^{Z^{\mathcal{A}}_a}_{E1}=\widetilde{G}_a\,G^{A}_{E1}$ contributes to a polarization observable there is also a leading-order contribution. The reason for this feature is that these terms arise from the interaction of the axial lepton current with the axial hadron current which is equivalent to the interaction of the lepton and hadron vector currents.

Finally, the tensor recoil polarizations offer another possibility of obtaining a clean access to P-violation via the axial form factors, *i.e.*

$$S_0\,A^{00\,+}_{ed}(20\,+) = -\frac{8\sqrt{2}}{3}\,g^d_v\,\widetilde{G}_a\left(G_C + \frac{\eta}{3}\,G_Q\right)G_Q$$
$$-\frac{\sqrt{2}}{3}\,\eta\left(1 + 2\,(1+\eta)\,\tan^2\frac{\theta}{2}\right)g^d_v\,\widetilde{G}_a\,G^2_M$$
$$-\frac{2\sqrt{2}}{3}\,\eta\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}$$
$$\times\left(G^{\gamma}_{E1} + \widetilde{G}_v\,G^A_{E1}\right)G_M, \tag{166}$$
$$S_0\,A^{00\,+}_{ed}(21\,+) =$$
$$\frac{8}{\sqrt{3}}\,\eta\,\sec\frac{\theta}{2}\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})}\,g^d_v\,\widetilde{G}_a\,G_M\,G_Q$$
$$+\frac{4}{\sqrt{3}}\,\eta\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G^{\gamma}_{E1} + \widetilde{G}_v\,G^A_{E1}\right)G_Q, \tag{167}$$
$$S_0\,A^{00\,+}_{ed}(22\,+) = -\frac{2}{\sqrt{3}}\,\eta\,g^d_v\,\widetilde{G}_a\,G^2_M\,. \tag{168}$$

5.3 T-violating contributions

Looking at the tables 3 through 5, one notes that T-violation induces very few nonvanishing observables. However, these appear always isolated, that means, they do not have to compete with leading order contributions or those from P-violation. The simplest candidate is the recoil vector polarization component P_y, perpendicular to the scattering plane [7–9], which is given by

$$S_0\,P_y = \frac{1}{\sqrt{3}}\,S_0\,A^{00\,+}_{d}(11\,-)$$
$$= \frac{4}{3}\,\sec\frac{\theta}{2}\,\eta\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G^{\gamma}_{E2}\,G_Q\,. \tag{169}$$

The latter result corresponds to the one given in [7,9] if one identifies the additional form factor G of [7,9] with $\frac{1}{2\,\eta}\,G^{\gamma}_{E2}$. With electron polarization one finds only one contribution from T-violation to the scalar asymmetries, namely to the tensor recoil polarization

$$S_0\,A^{00\,+}_{ed}(21\,-) =$$
$$\frac{4}{\sqrt{3}}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\,G^{\gamma}_{E2}\left(G_C + \frac{1}{3}\,\eta\,G_Q\right). \tag{170}$$

With this we will conclude the formal study of polarization observables in elastic electron deuteron scattering.

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Appendix A. Symmetries and closed form of $\mathcal{U}^{\lambda'\lambda\,IM}_{I'M'}(c',c)$

Here we will derive the various symmetries listed in (82) through (84). We will start by considering first the symmetries of the t-matrix elements given in (53). For the reduced multipole matrix elements one finds as symmetry properties

$$(\mathcal{O}^{\lambda}_L(c))^* = (-)^{\lambda}\,\mathcal{O}^{\lambda}_L(c)\,, \tag{A.1}$$
$$\mathcal{O}^{\lambda}_L(c) = (-)^{L+\delta^P_c}\,\mathcal{O}^{-\lambda}_L(c)\,, \tag{A.2}$$
$$\mathcal{O}^{\lambda}_L(c) = (-)^{L+\lambda+\delta^T_c}\,\mathcal{O}^{\lambda}_L(c)\,, \tag{A.3}$$

which follow from hermiticity, and from parity and time reversal transformations, respectively. First we note

$$(t^c_{m'\lambda m})^* =$$
$$(-)^{1-m'+\lambda}\,a_\lambda\sum_L i^L(-)^L\hat{L}\begin{pmatrix}1 & L & 1\\ -m' & \lambda & m\end{pmatrix}(\mathcal{O}^{\lambda}_L(c))^*. \tag{A.4}$$

Using hermiticity and time reversal properties from (A.1) and (A.3), yielding $(\mathcal{O}^{\lambda}_L(c))^* = (-)^{L+\delta^T_c}\,\mathcal{O}^{\lambda}_L(c)$, one finds

$$(t^c_{m'\lambda m})^* = (-)^{\delta^T_c}\,t^c_{m'\lambda m}\,, \tag{A.5}$$

which means that all t-matrix elements are real or imaginary quantities depending on whether $(-)^{\delta^T_c} = \pm 1$, respectively. From this relation and the fact that the matrix elements of the statistical tensors are real follows directly (81), which means that the $\mathcal{U}$'s are real or imaginary depending on whether $(-)^{\delta^T_c+\delta^T_{c'}} = \pm 1$, respectively. Second we consider

$$t^c_{-m'-\lambda-m} = (-)^{1+m'-\lambda}\,a_\lambda\sum_L i^L\hat{L}\begin{pmatrix}1 & L & 1\\ m' & -\lambda & -m\end{pmatrix}\mathcal{O}^{-\lambda}_L(c)$$
$$= \left((-)^{1-m'+\lambda}\,a_\lambda\sum_L i^L\hat{L}\begin{pmatrix}1 & L & 1\\ -m' & \lambda & m\end{pmatrix}(\mathcal{O}^{-\lambda}_L(c))^*\right)^*, \tag{A.6}$$

where in the second expression we have made use of the symmetry of the $3j$-symbol. This then gives the relation

$$t^c_{-m'-\lambda-m} = (-)^{\delta_c}\,(t^c_{m'\lambda m})^* = (-)^{\delta^P_c}\,t^c_{m'\lambda m}\,, \tag{A.7}$$

using $(\mathcal{O}^{-\lambda}_L(c))^* = (-)^{\delta_c}\,\mathcal{O}^{\lambda}_L(c)$ from (A.2) and (A.3). The same relation can be applied to (A.4) together with the symmetry of the $3j$-symbol with respect to a sign change of all projections, resulting in

$$(t^c_{m'\lambda m})^* = (-)^{\delta^P_c}\,t^c_{m-\lambda m'}\,. \tag{A.8}$$

Now we are ready to prove the symmetries of the $\mathcal{U}^{\lambda'\lambda\,IM}_{I'M'}(c',c)$. First we consider the interchange $\lambda \leftrightarrow \lambda'$ which gives

$$\mathcal{U}^{\lambda\lambda'\,IM}_{I'M'}(c',c) = \frac{1}{6}$$
$$\times\sum_{n',n,m',m}\left(t^{c'\,*}_{n'\lambda n}(\tau^{[I']}_{M'})_{n'm'}\,t^c_{m'\lambda' m}\,(\tau^{[I]}_{M})_{mn} + (c'\leftrightarrow c)\right). \tag{A.9}$$

Using

$$(\tau^{[I]}_{M})_{mn} = (-)^M(\tau^{[I]}_{-M})_{nm} \tag{A.10}$$

and renaming the indices, one obtains the symmetry of (82)

$$\mathcal{U}_{I'M'}^{\lambda\lambda' IM}(c',c) = (-)^{M'+M}\Big(\mathcal{U}_{I'-M'}^{\lambda'\lambda\, I-M}(c',c)\Big)^* \tag{A.11}$$

$$= (-)^{\lambda'+\lambda}\mathcal{U}_{I'-M'}^{\lambda'\lambda\, I-M}(c',c)\,, \tag{A.12}$$

where the latter follows from (80) and (81). The second symmetry refers to the sign change of the various projections

$$\mathcal{U}_{I'-M'}^{-\lambda'-\lambda\, I-M}(c',c) = \frac{1}{6} \times \sum_{n',n,m',m}\Big(t_{n'-\lambda'n}^{c'\,*}(\tau_{-M'}^{[I']})_{n'm'}\, t_{m'-\lambda m}^{c}\,(\tau_{-M}^{[I]})_{mn} + (c'\leftrightarrow c)\Big). \tag{A.13}$$

Changing the signs of all summation indices, using (A.7) and the property

$$(\tau_{-M}^{[I]})_{-m-n} = (-)^I(\tau_M^{[I]})_{mn} \tag{A.14}$$

results in (83). Finally, considering

$$\mathcal{U}_{IM}^{\lambda'\lambda\, I'M'}(c',c) = \frac{1}{6} \times \sum_{n',n,m',m}\Big(t_{n'\lambda'n}^{c'*}\,(\tau_M^{[I]})_{n'm'} t_{m'\lambda m}^{c}\,(\tau_{M'}^{[I']})_{mn} + (c'\leftrightarrow c)\Big), \tag{A.15}$$

making for the summation indices the interchanges $m' \leftrightarrow m$ and $n' \leftrightarrow n$, using (A.10) and (A.7), one first finds

$$\mathcal{U}_{IM}^{\lambda'\lambda\, I'M'}(c',c) = (-)^{\delta^{PT}(c',c)+I+I'}\mathcal{U}_{I'-M'}^{-\lambda'-\lambda\, I-M}(c',c)\,, \tag{A.16}$$

which gives combined with (83) the symmetry of (84).

At the end of this appendix, we will derive a closed expression for $\mathcal{U}_{IM}^{\lambda'\lambda\, I'M'}(c',c)$ in terms of reduced multipole matrix elements. To this end we use the multipole expansion of the t-matrix and the Wigner-Eckart theorem for the occurring matrix elements of the multipole operators and statistical tensors

$$\langle 1m'|\mathcal{O}_{LM}^{\lambda}|1m\rangle = (-)^{1-m'}\begin{pmatrix} 1 & L & 1 \\ -m' & M & m\end{pmatrix}\mathcal{O}_L^{\lambda}\,, \tag{A.17}$$

$$\langle 1m'|\tau_M^{[I]}|1m\rangle = (-)^{1-m'}\begin{pmatrix} 1 & I & 1 \\ -m' & M & m\end{pmatrix}\sqrt{3}\,\hat{I}\,. \tag{A.18}$$

With the help of a sum rule for a sum over four $3j$-symbols [13]

$$\mathcal{S}\begin{bmatrix} L & L' & I & I' \\ \lambda & \lambda' & M & M'\end{bmatrix} = \sum_{n',n,m',m}(-)^{\lambda+L'+I'+m'+m}\begin{pmatrix} 1 & L & 1 \\ -m' & \lambda & m\end{pmatrix}\begin{pmatrix} 1 & L' & 1 \\ -n' & \lambda' & n\end{pmatrix} \times \begin{pmatrix} 1 & I & 1 \\ -m & M & n\end{pmatrix}\begin{pmatrix} 1 & I' & 1 \\ -n' & M' & m'\end{pmatrix}$$

$$= \sum_{J,m}\hat{J}^2\begin{pmatrix} L & L' & J \\ \lambda & -\lambda' & m\end{pmatrix}\begin{pmatrix} I & I' & J \\ -M & -M' & m\end{pmatrix}\begin{Bmatrix} L & L' & J \\ 1 & 1 & I \\ 1 & 1 & I'\end{Bmatrix}, \tag{A.19}$$

one obtains in closed form

$$\mathcal{U}_{I'M'}^{\lambda'\lambda\, IM}(c',c) = (-)^{\lambda'+I'}\frac{1}{2}a_{\lambda'}\,a_{\lambda}\,\hat{I}\,\hat{I}'\sum_{L',L} i^{L'+L}\,\hat{L}\,\hat{L}'\mathcal{S} \times \begin{bmatrix} L & L' & I & I' \\ \lambda & \lambda' & M & M'\end{bmatrix}\Big(\mathcal{O}_{L'}^{\lambda'\,*}(c')\,\mathcal{O}_L^{\lambda}(c) + (c'\leftrightarrow c)\Big)\,. \tag{A.20}$$

Appendix B. General expressions for the f-functions

Here we list all nonvanishing f-functions for the case of recoil polarization without target polarization, *i.e.*, $f_\alpha^{00+}(I'M'\mathrm{sig}_{M'};c',c)$ for the various diagonal and interference contributions.

(A) Diagonal contributions:

(i) $c',\ c \in \mathcal{C}_{pc,\,tc}$:

$$f_L^{00+}(00+;c',c) = \frac{4\pi}{3}\Big(C_0(c)\,C_0(c') + C_2(c)\,C_2(c')\Big), \tag{B.1}$$

$$f_L^{00+}(20+;\,c',c) = -\frac{2\pi}{3}\Big(2\,C_0(c')\,C_2(c) + 2\,C_0(c)\,C_2(c') + \sqrt{2}\,C_2(c)\,C_2(c')\Big), \tag{B.2}$$

$$f_T^{00+}(00+;\,c',c) = \frac{4\pi}{3}\,M_1(c)\,M_1(c'), \tag{B.3}$$

$$f_T^{00+}(20+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,M_1(c)\,M_1(c'), \tag{B.4}$$

$$f_T'^{\,00+}(10+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,M_1(c)\,M_1(c'), \tag{B.5}$$

$$f_{LT}^{00+}(21+;c',c) = 2\pi\Big(C_2(c')\,M_1(c) + C_2(c)\,M_1(c')\Big), \tag{B.6}$$

$$f_{LT}'^{\,00+}(11+;\,c',c) = \frac{2\pi}{3}\Big((2\sqrt{2}\,C_0(c') + C_2(c'))\,M_1(c) + (2\sqrt{2}\,C_0(c) + C_2(c))\,M_1(c')\Big), \tag{B.7}$$

$$f_{TT}^{00+}(22+;\,c',c) = \frac{2\pi}{\sqrt{3}}\,M_1(c)\,M_1(c'). \tag{B.8}$$

(ii) $c',\ c \in \mathcal{C}_{pc,\,tnc}$:

$$f_T^{00+}(00+;\,c',c) = \frac{4\pi}{3}\,E_2(c)\,E_2(c'), \tag{B.9}$$

$$f_T^{00+}(20+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,E_2(c)\,E_2(c'), \tag{B.10}$$

$$f_T'^{\,00+}(10+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,E_2(c)\,E_2(c'), \tag{B.11}$$

$$f_{TT}^{00+}(22+;\,c',c) = \frac{2\pi}{\sqrt{3}}\,E_2(c)\,E_2(c'). \tag{B.12}$$

(iii) $c',\ c \in \mathcal{C}_{pnc,\,tc}$:

$$f_T^{00\,+}(00\,+;\,c',c) = \frac{4\,\pi}{3}\,E_1(c)\,E_1(c'), \tag{B.13}$$

$$f_T^{00\,+}(20\,+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,E_1(c)\,E_1(c'), \tag{B.14}$$

$$f_T'^{\,00\,+}(10\,+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,E_1(c)\,E_1(c'), \tag{B.15}$$

$$f_{TT}^{00\,+}(22\,+;\,c',c) = -\frac{2\,\pi}{\sqrt{3}}\,E_1(c)\,E_1(c'). \tag{B.16}$$

(iv) $c',\,c \in \mathcal{C}_{pnc,\,tnc}$:

$$f_L^{00\,+}(00\,+;\,c',c) = \frac{4\,\pi}{3}\,C_1(c)\,C_1(c'), \tag{B.17}$$

$$f_L^{00\,+}(20\,+;\,c',c) = \frac{2\,\sqrt{2}\,\pi}{3}\,C_1(c)\,C_1(c'), \tag{B.18}$$

$$f_T^{00\,+}(00\,+;\,c',c) = \frac{4\,\pi}{3}\,M_2(c)\,M_2(c'), \tag{B.19}$$

$$f_T^{00\,+}(20\,+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,M_2(c)\,M_2(c'), \tag{B.20}$$

$$f_T'^{\,00\,+}(10\,+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,M_2(c)\,M_2(c'), \tag{B.21}$$

$$f_{LT}^{00+}(21+;c',c) = -\frac{2\pi}{\sqrt{3}}\Big(C_1(c')M_2(c)+C_1(c)M_2(c')\Big), \tag{B.22}$$

$$f_{LT}'^{\,00+}(11+;c',c) = \frac{2\pi}{\sqrt{3}}\Big(C_1(c')M_2(c)+C_1(c)M_2(c')\Big), \tag{B.23}$$

$$f_{TT}^{00\,+}(22\,+;\,c',c) = -\frac{2\,\pi}{\sqrt{3}}\,M_2(c)\,M_2(c'). \tag{B.24}$$

(B) Interference contributions:

(i) $c' \in \mathcal{C}_{pc,\,tc}$ and $c \in \mathcal{C}_{pc,\,tnc}$:

$$f_{LT}^{00\,+}(11\,-;\,c',c) = 2\,\pi\,C_2(c')\,E_2(c), \tag{B.25}$$

$$f_{LT}'^{\,00+}(21-;\,c',c) = \frac{2\pi}{3}\Big(2\sqrt{2}C_0(c')+C_2(c')\Big)E_2(c). \tag{B.26}$$

(ii) $c' \in \mathcal{C}_{pc,\,tc}$ and $c \in \mathcal{C}_{pnc,\,tc}$:

$$f_T^{00\,+}(10\,+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,E_1(c)\,M_1(c'), \tag{B.27}$$

$$f_T'^{\,00\,+}(00\,+;\,c',c) = \frac{4\,\pi}{3}\,E_1(c)\,M_1(c'), \tag{B.28}$$

$$f_T'^{\,00\,+}(20\,+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,E_1(c)\,M_1(c'), \tag{B.29}$$

$$f_{LT}^{00+}(11+;\,c',c) = \frac{2\pi}{3}\Big(2\sqrt{2}C_0(c')+C_2(c')\Big)E_1(c), \tag{B.30}$$

$$f_{LT}'^{\,00\,+}(21\,+;\,c',c) = 2\,\pi\,C_2(c')\,E_1(c). \tag{B.31}$$

(iii) $c' \in \mathcal{C}_{pc,\,tc}$ and $c \in \mathcal{C}_{pnc,\,tnc}$:

$$f_{LT}^{00\,+}(21\,-;\,c',c) = -\frac{2\,\pi}{3}\,\Big(\sqrt{3}\,C_1(c)\,M_1(c') - \Big(2\,\sqrt{2}\,C_0(c') + C_2(c')\Big)\,M_2(c)\Big), \tag{B.32}$$

$$f_{LT}'^{\,00\,+}(11\,-;\,c',c) = \frac{2\,\pi}{3}\,\Big(\sqrt{3}\,C_1(c)\,M_1(c') + 3\,C_2(c')\,M_2(c)\Big), \tag{B.33}$$

$$f_{TT}^{00\,+}(22\,-;\,c',c) = \frac{2\,\pi}{\sqrt{3}}\,M_1(c')\,M_2(c). \tag{B.34}$$

(iv) $c' \in \mathcal{C}_{pc,\,tnc}$ and $c \in \mathcal{C}_{pnc,\,tc}$:

$$f_{TT}^{00\,+}(22\,-;\,c',c) = -\frac{2\,\pi}{\sqrt{3}}\,E_1(c)\,E_2(c'). \tag{B.35}$$

(v) $c' \in \mathcal{C}_{pc,\,tnc}$ and $c \in \mathcal{C}_{pnc,\,tnc}$:

$$f_T^{00\,+}(10\,+;\,c',c) = \sqrt{\frac{2}{3}}\,\pi\,E_2(c')\,M_2(c), \tag{B.36}$$

$$f_T'^{\,00\,+}(00\,+;\,c',c) = \frac{4\,\pi}{3}\,E_2(c')\,M_2(c), \tag{B.37}$$

$$f_T'^{\,00\,+}(20\,+;\,c',c) = -\frac{\sqrt{2}\,\pi}{3}\,E_2(c')\,M_2(c), \tag{B.38}$$

$$f_{LT}^{00\,+}(11\,+;\,c',c) = \frac{2\,\pi}{\sqrt{3}}\,C_1(c)\,E_2(c'), \tag{B.39}$$

$$f_{LT}'^{\,00\,+}(21\,+;\,c',c) = -\frac{2\,\pi}{\sqrt{3}}\,C_1(c)\,E_2(c'). \tag{B.40}$$

(vi) $c' \in \mathcal{C}_{pnc,\,tc}$ and $c \in \mathcal{C}_{pnc,\,tnc}$:

$$f_{LT}^{00\,+}(11\,-;\,c',c) = \frac{2\,\pi}{\sqrt{3}}\,C_1(c)\,E_1(c'), \tag{B.41}$$

$$f_{LT}'^{\,00\,+}(21\,-;\,c',c) = -\frac{2\,\pi}{\sqrt{3}}\,C_1(c)\,E_1(c'). \tag{B.42}$$

Appendix C. Listing of structure functions including P- and T-violation

Here we list all nonvanishing structure functions $F_\alpha^{IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'})$ and $\widetilde{F}_\alpha^{IM\mathrm{sig}_M}(I'M'\mathrm{sig}_{M'})$ for $\mathrm{sig}_M = +$, $I' \geq I$, and $M' \geq M$. Those for $I' < I$, and $M' < M$ as well as the ones for $\mathrm{sig}_M = -$ can be obtained from the listed ones using the symmetry relations in (100), (104) and (105). Note that $\mathrm{sig}_{M'}$ is fixed uniquely with the choice of sig_M.

(i) P- and T-conserved structure functions:

$$F_L^{00+}(00+) = \frac{4\pi}{3}\left((C_0^\gamma)^2 + (C_2^\gamma)^2\right), \tag{C.1}$$

$$F_L^{00+}(20+) = -\frac{2\pi}{3}C_2^\gamma\left(4C_0^\gamma + \sqrt{2}C_2^\gamma\right), \tag{C.2}$$

$$F_L^{10+}(10+) = \frac{2\pi}{3}\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right)^2, \tag{C.3}$$

$$F_L^{11+}(11+) = \frac{4\pi}{3}\left(2(C_0^\gamma)^2 + \sqrt{2}C_0^\gamma C_2^\gamma - 2(C_2^\gamma)^2\right), \tag{C.4}$$

$$F_L^{20+}(20+) = \frac{2\pi}{3}\left(2(C_0^\gamma)^2 + 2\sqrt{2}C_0^\gamma C_2^\gamma + 3(C_2^\gamma)^2\right), \tag{C.5}$$

$$F_L^{21+}(21+) = \frac{4\pi}{3}\left(2(C_0^\gamma)^2 + \sqrt{2}C_0^\gamma C_2^\gamma - 2(C_2^\gamma)^2\right), \tag{C.6}$$

$$F_L^{22+}(22+) = \frac{4\pi}{3}\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right)^2, \tag{C.7}$$

$$F_T^{00+}(00+) = \frac{4\pi}{3}(M_1^\gamma)^2, \tag{C.8}$$

$$F_T^{00+}(20+) = -\frac{\sqrt{2}\pi}{3}(M_1^\gamma)^2, \tag{C.9}$$

$$F_T^{11+}(11+) = 2\pi(M_1^\gamma)^2, \tag{C.10}$$

$$F_T^{20+}(20+) = -\frac{4\pi}{3}(M_1^\gamma)^2, \tag{C.11}$$

$$F_T^{21+}(21+) = -2\pi(M_1^\gamma)^2, \tag{C.12}$$

$$F_T'^{00+}(10+) = \sqrt{\frac{2}{3}}\pi(M_1^\gamma)^2, \tag{C.13}$$

$$F_T'^{10+}(20+) = \frac{2\pi}{\sqrt{3}}(M_1^\gamma)^2, \tag{C.14}$$

$$F_T'^{11+}(21+) = 2\pi(M_1^\gamma)^2, \tag{C.15}$$

$$F_{LT}^{00+}(21+) = 4\pi C_2^\gamma M_1^\gamma, \tag{C.16}$$

$$F_{LT}^{10+}(11+) = \frac{2\pi}{\sqrt{3}}\left(-2C_0^\gamma + \sqrt{2}C_2^\gamma\right)M_1^\gamma, \tag{C.17}$$

$$F_{LT}^{20+}(21+) = -2\pi\left(2C_0^\gamma + \sqrt{2}C_2^\gamma\right)M_1^\gamma, \tag{C.18}$$

$$F_{LT}^{21+}(22+) = 2\sqrt{\frac{2}{3}}\pi\left(-2C_0^\gamma + \sqrt{2}C_2^\gamma\right)M_1^\gamma, \tag{C.19}$$

$$F_{LT}'^{00+}(11+) = \frac{4\pi}{3}\left(2\sqrt{2}C_0^\gamma + C_2^\gamma\right)M_1^\gamma, \tag{C.20}$$

$$F_{LT}'^{10+}(21+) = 2\sqrt{\frac{2}{3}}\pi\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right)M_1^\gamma, \tag{C.21}$$

$$F_{LT}'^{11+}(20+) = -\frac{2\pi}{3}\left(2C_0^\gamma + 5\sqrt{2}C_2^\gamma\right)M_1^\gamma, \tag{C.22}$$

$$F_{LT}'^{11+}(22+) = -2\sqrt{\frac{2}{3}}\pi\left(-2C_0^\gamma + \sqrt{2}C_2^\gamma\right)M_1^\gamma, \tag{C.23}$$

$$F_{TT}^{00+}(22+) = \frac{2\pi}{\sqrt{3}}(M_1^\gamma)^2, \tag{C.24}$$

$$F_{TT}^{11+}(11+) = 2\pi(M_1^\gamma)^2, \tag{C.25}$$

$$F_{TT}^{20+}(22+) = -2\sqrt{\frac{2}{3}}\pi(M_1^\gamma)^2, \tag{C.26}$$

$$F_{TT}^{21+}(21+) = 2\pi(M_1^\gamma)^2. \tag{C.27}$$

(ii) P-violating structure functions:

$$F_T^{00+}(10+) = 2\sqrt{\frac{2}{3}}\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.28}$$

$$F_T^{10+}(20+) = \frac{4\pi}{\sqrt{3}}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.29}$$

$$F_T^{11+}(21+) = 4\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.30}$$

$$F_T'^{00+}(00+) = \frac{8\pi}{3}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.31}$$

$$F_T'^{00+}(20+) = -2\frac{\sqrt{2}\pi}{3}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.32}$$

$$F_T'^{11+}(11+) = 4\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.33}$$

$$F_T'^{20+}(20+) = -\frac{8\pi}{3}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.34}$$

$$F_T'^{21+}(21+) = -4\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)M_1^\gamma, \tag{C.35}$$

$$F_{LT}^{00+}(11+) = \frac{4\pi}{3}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(2\sqrt{2}C_0^\gamma + C_2^\gamma\right), \tag{C.36}$$

$$F_{LT}^{10+}(21+) = \frac{4\pi}{\sqrt{6}}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right), \tag{C.37}$$

$$F_{LT}^{11+}(20+) = -\frac{2\pi}{3}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(2C_0^\gamma + 5\sqrt{2}C_2^\gamma\right), \tag{C.38}$$

$$F_{LT}^{11+}(22+) = \frac{4\pi}{\sqrt{3}}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right), \tag{C.39}$$

$$F_{LT}'^{00+}(21+) = 4\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)C_2^\gamma, \tag{C.40}$$

$$F_{LT}'^{10+}(11+) = -\frac{4\pi}{\sqrt{6}}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right), \tag{C.41}$$

$$F_{LT}'^{20+}(21+) = -2\sqrt{2}\pi\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(\sqrt{2}C_0^\gamma + C_2^\gamma\right), \tag{C.42}$$

$$F_{LT}'^{21+}(22+) = -\frac{4\pi}{\sqrt{3}}\left(E_1^\gamma + E_1^{Z_a^{\mathcal{A}}}\right)\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right). \tag{C.43}$$

(iii) T-violating structure functions:

$$F_T^{11+}(21-) = 4\pi E_2^\gamma M_1^\gamma, \tag{C.44}$$

$$F_T'^{11+}(11-) = 4\pi E_2^\gamma M_1^\gamma, \tag{C.45}$$

$$F_T'^{21+}(21-) = -4\pi E_2^\gamma M_1^\gamma, \tag{C.46}$$

$$F_{LT}^{00+}(11-) = 4\pi\, C_2^\gamma\, E_2^\gamma, \tag{C.47}$$
$$F_{LT}^{10+}(21-) = -\frac{4\pi}{\sqrt{6}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma, \tag{C.48}$$
$$F_{LT}^{11+}(22-) = \frac{4\pi}{\sqrt{3}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma, \tag{C.49}$$

$$F_{LT}'^{00+}(21-) = \frac{4\pi}{3}\left(2\sqrt{2}\,C_0^\gamma + C_2^\gamma\right) E_2^\gamma, \tag{C.50}$$
$$F_{LT}'^{10+}(11-) = \frac{4\pi}{\sqrt{6}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma, \tag{C.51}$$
$$F_{LT}'^{20+}(21-) = -\frac{2\sqrt{2}}{3}\,\pi\left(\sqrt{2}\,C_0^\gamma + 5\,C_2^\gamma\right) E_2^\gamma, \tag{C.52}$$
$$F_{LT}'^{21+}(22-) = -\frac{4\pi}{\sqrt{3}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma, \tag{C.53}$$

$$F_{TT}^{11+}(21-) = 4\pi\, E_2^\gamma\, M_1^\gamma. \tag{C.54}$$

(iv) P-violating structure functions $\widetilde{F}$:

$$\widetilde{F}_L^{00+}(00+) = \frac{8\pi}{3}\left(C_0^\gamma\, C_0^{Z_v^{\mathcal{A}}} + C_2^\gamma\, C_2^{Z_v^{\mathcal{A}}}\right), \tag{C.55}$$
$$\widetilde{F}_L^{00+}(20+) = -\frac{4\pi}{3}\left(2\,C_0^{Z_v^{\mathcal{A}}}\, C_2^\gamma + \left(2\,C_0^\gamma + \sqrt{2}\,C_2^\gamma\right) C_2^{Z_v^{\mathcal{A}}}\right), \tag{C.56}$$
$$\widetilde{F}_L^{10+}(10+) = \frac{4\pi}{3}\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right)\left(\sqrt{2}C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\right), \tag{C.57}$$
$$\widetilde{F}_L^{11+}(11+) = \frac{4\pi}{3}\Big(C_2^\gamma\left(\sqrt{2}C_0^{Z_v^{\mathcal{A}}} - 4C_2^{Z_v^{\mathcal{A}}}\right) + C_0^\gamma\left(4C_0^{Z_v^{\mathcal{A}}} + \sqrt{2}C_2^{Z_v^{\mathcal{A}}}\right)\Big), \tag{C.58}$$
$$\widetilde{F}_L^{20+}(20+) = \frac{4\pi}{3}\Big(C_2^\gamma\left(\sqrt{2}C_0^{Z_v^{\mathcal{A}}} + 3C_2^{Z_v^{\mathcal{A}}}\right) + C_0^\gamma\left(2C_0^{Z_v^{\mathcal{A}}} + \sqrt{2}C_2^{Z_v^{\mathcal{A}}}\right)\Big), \tag{C.59}$$
$$\widetilde{F}_L^{21+}(21+) = \frac{4\pi}{3}\Big(C_2^\gamma\left(\sqrt{2}C_0^{Z_v^{\mathcal{A}}} - 4C_2^{Z_v^{\mathcal{A}}}\right) + C_0^\gamma\left(4C_0^{Z_v^{\mathcal{A}}} + \sqrt{2}C_2^{Z_v^{\mathcal{A}}}\right)\Big), \tag{C.60}$$
$$\widetilde{F}_L^{22+}(22+) = \frac{8\pi}{3}\left(\sqrt{2}C_0^\gamma - C_2^\gamma\right)\left(\sqrt{2}C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\right), \tag{C.61}$$

$$\widetilde{F}_T^{00+}(00+) = \frac{8\pi}{3}\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.62}$$
$$\widetilde{F}_T^{00+}(10+) = 2\sqrt{\frac{2}{3}}\,\pi\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.63}$$
$$\widetilde{F}_T^{00+}(20+) = -\frac{2\sqrt{2}}{3}\,\pi\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.64}$$
$$\widetilde{F}_T^{10+}(20+) = \frac{4\pi}{\sqrt{3}}\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.65}$$
$$\widetilde{F}_T^{11+}(11+) = 4\pi\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.66}$$
$$\widetilde{F}_T^{11+}(21+) = 4\pi\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.67}$$
$$\widetilde{F}_T^{20+}(20+) = -\frac{8\pi}{3}\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.68}$$
$$\widetilde{F}_T^{21+}(21+) = -4\pi\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.69}$$

$$\widetilde{F}_T'^{00+}(00+) = \frac{8\pi}{3}\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.70}$$
$$\widetilde{F}_T'^{00+}(10+) = 2\sqrt{\frac{2}{3}}\,\pi\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.71}$$
$$\widetilde{F}_T'^{00+}(20+) = -\frac{2\sqrt{2}\,\pi}{3}\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.72}$$
$$\widetilde{F}_T'^{10+}(20+) = \frac{4\pi}{\sqrt{3}}\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.73}$$
$$\widetilde{F}_T'^{11+}(11+) = 4\pi\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.74}$$
$$\widetilde{F}_T'^{11+}(21+) = 4\pi\, M_1^\gamma\, M_1^{Z_v^{\mathcal{A}}}, \tag{C.75}$$
$$\widetilde{F}_T'^{20+}(20+) = -\frac{8\pi}{3}\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.76}$$
$$\widetilde{F}_T'^{21+}(21+) = -4\pi\, E_1^{Z_a^{\mathcal{A}}}\, M_1^\gamma, \tag{C.77}$$

$$\widetilde{F}_{LT}^{00+}(11+) = \frac{4\pi}{3}\left(2\sqrt{2}\,C_0^\gamma + C_2^\gamma\right) E_1^{Z_a^{\mathcal{A}}}, \tag{C.78}$$
$$\widetilde{F}_{LT}^{00+}(21+) = 4\pi\left(C_2^{Z_v^{\mathcal{A}}}\, M_1^\gamma + C_2^\gamma\, M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.79}$$
$$\widetilde{F}_{LT}^{10+}(11+) = -2\sqrt{\frac{2}{3}}\,\pi\left(\left(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.80}$$
$$\widetilde{F}_{LT}^{10+}(21+) = \frac{4\pi}{\sqrt{6}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_1^{Z_a^{\mathcal{A}}}, \tag{C.81}$$
$$\widetilde{F}_{LT}^{11+}(20+) = -\frac{2\pi}{3}\left(2\,C_0^\gamma + 5\sqrt{2}\,C_2^\gamma\right) E_1^{Z_a^{\mathcal{A}}}, \tag{C.82}$$
$$\widetilde{F}_{LT}^{11+}(22+) = \frac{4\pi}{\sqrt{6}}\left(2\,C_0^\gamma - \sqrt{2}\,C_2^\gamma\right) E_1^{Z_a^{\mathcal{A}}}, \tag{C.83}$$
$$\widetilde{F}_{LT}^{20+}(21+) = -2\sqrt{2}\,\pi\left(\left(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} + C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(\sqrt{2}\,C_0^\gamma + C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.84}$$
$$\widetilde{F}_{LT}^{21+}(22+) = -\frac{4\pi}{\sqrt{3}}\left(\left(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.85}$$

$$\widetilde{F}_{LT}'^{00+}(11+) = \frac{2\sqrt{2}}{3}\,\pi\left(\left(4\,C_0^{Z_v^{\mathcal{A}}} + \sqrt{2}\,C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(4\,C_0^\gamma + \sqrt{2}\,C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.86}$$
$$\widetilde{F}_{LT}'^{00+}(21+) = 4\pi\, C_2^\gamma\, E_1^{Z_a^{\mathcal{A}}}, \tag{C.87}$$
$$\widetilde{F}_{LT}'^{10+}(11+) = -\frac{4\pi}{\sqrt{6}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_1^{Z_a^{\mathcal{A}}}, \tag{C.88}$$
$$\widetilde{F}_{LT}'^{10+}(21+) = 2\sqrt{\frac{2}{3}}\,\pi\left(\left(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.89}$$
$$\widetilde{F}_{LT}'^{11+}(20+) = -\frac{2\sqrt{2}}{3}\,\pi\left(\left(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} + 5\,C_2^{Z_v^{\mathcal{A}}}\right) M_1^\gamma + \left(\sqrt{2}\,C_0^\gamma + 5\,C_2^\gamma\right) M_1^{Z_v^{\mathcal{A}}}\right), \tag{C.90}$$

$$\widetilde{F}_{LT}^{\prime\,11\,+}(22\,+) = \frac{4\,\pi}{\sqrt{3}}\Big(\Big(\sqrt{2}\,C_0^{Z_v^{\mathcal{A}}} - C_2^{Z_v^{\mathcal{A}}}\Big)\,M_1^\gamma + \Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)\,M_1^{Z_v^{\mathcal{A}}}\Big), \tag{C.91}$$

$$\widetilde{F}_{LT}^{\prime\,20\,+}(21\,+) = -2\,\sqrt{2}\,\pi\,\Big(\sqrt{2}\,C_0^\gamma + C_2^\gamma\Big)\,E_1^{Z_a^{\mathcal{A}}}, \tag{C.92}$$

$$\widetilde{F}_{LT}^{\prime\,21\,+}(22\,+) = -\frac{4\,\pi}{\sqrt{3}}\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)\,E_1^{Z_a^{\mathcal{A}}}, \tag{C.93}$$

$$\widetilde{F}_{TT}^{00\,+}(22\,+) = \frac{4\,\pi}{\sqrt{3}}\,M_1^\gamma\,M_1^{Z_v^{\mathcal{A}}}, \tag{C.94}$$

$$\widetilde{F}_{TT}^{11\,+}(11\,+) = 4\,\pi\,M_1^\gamma\,M_1^{Z_v^{\mathcal{A}}}, \tag{C.95}$$

$$\widetilde{F}_{TT}^{20\,+}(22\,+) = -4\,\sqrt{\frac{2}{3}}\,\pi\,M_1^\gamma\,M_1^{Z_v^{\mathcal{A}}}, \tag{C.96}$$

$$\widetilde{F}_{TT}^{21\,+}(21\,+) = 4\,\pi\,M_1^\gamma\,M_1^{Z_v^{\mathcal{A}}}. \tag{C.97}$$

Appendix D. Listing of various asymmetries

Here we list all nonvanishing asymmetries $A_{d/ed}^{IM\mathrm{sig}_M}\cdot(I'M'\mathrm{sig}_{M'})$ for $\mathrm{sig}_M = +$, $I' \geq I$, and $M' \geq M$. Those for $I' < I$, and $M' < M$ as well as the ones for $\mathrm{sig}_M = -$ can be obtained from the listed ones using the symmetry relations in (100), (132) and (133). Note that $\mathrm{sig}_{M'}$ is fixed uniquely with the choice of sig_M.

(A) Asymmetries for P- and T-conserved contributions:

(i) Scalar asymmetries:

$$S_0\,A_d^{00\,+}(00\,+) = \frac{4\,\pi}{3}\,\Big((C_0^\gamma)^2 + (C_2^\gamma)^2\Big)\,v_L + \frac{4\,\pi}{3}\,(M_1^\gamma)^2\,v_T\,, \tag{D.1}$$

$$S_0\,A_d^{00\,+}(20\,+) = -\frac{2\,\pi}{3}\,\Big(4\,C_0^\gamma\sqrt{2}\,C_2^\gamma\Big)\,C_2^\gamma\,v_L - \frac{\sqrt{2}\,\pi}{3}\,(M_1^\gamma)^2\,v_T\,, \tag{D.2}$$

$$S_0\,A_d^{00\,+}(21\,+) = 4\,\pi\,C_2^\gamma\,M_1^\gamma\,v_{LT}\,, \tag{D.3}$$

$$S_0\,A_d^{00\,+}(22\,+) = \frac{2\,\pi}{\sqrt{3}}\,(M_1^\gamma)^2\,v_{TT}\,, \tag{D.4}$$

$$S_0\,A_{ed}^{00\,+}(10\,+) = \sqrt{\frac{2}{3}}\,\pi\,(M_1^\gamma)^2\,v_T'\,, \tag{D.5}$$

$$S_0\,A_{ed}^{00\,+}(11\,+) = \frac{2\,\sqrt{2}\,\pi}{3}\Big(4\,C_0^\gamma + \sqrt{2}\,C_2^\gamma\Big)\,M_1^\gamma\,v_{LT}'. \tag{D.6}$$

(ii) Vector asymmetries:

$$S_0\,A_d^{10\,+}(10\,+) = \frac{2\,\pi}{3}\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)^2\,v_L\,, \tag{D.7}$$

$$S_0\,A_d^{10\,+}(11\,+) = -2\,\sqrt{\frac{2}{3}}\,\pi\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\,\Big)M_1^\gamma\,v_{LT}\,, \tag{D.8}$$

$$S_0\,A_d^{11\,+}(11\,+) = \frac{4\pi}{3}\Big(2\,(C_0^\gamma)^2 + \sqrt{2}\,C_0^\gamma C_2^\gamma - 2\,(C_2^\gamma)^2\Big)\,v_L + 2\,\pi\,(M_1^\gamma)^2\,(v_T + v_{TT})\,, \tag{D.9}$$

$$S_0\,A_{ed}^{10\,+}(20\,+) = \frac{2\,\pi}{\sqrt{3}}\,(M_1^\gamma)^2\,v_T'\,, \tag{D.10}$$

$$S_0\,A_{ed}^{10\,+}(21\,+) = 2\,\sqrt{\frac{2}{3}}\,\pi\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)\,M_1^\gamma\,v_{LT}'\,, \tag{D.11}$$

$$S_0\,A_{ed}^{11\,+}(21\,+) = 2\,\pi\,(M_1^\gamma)^2\,v_T'\,, \tag{D.12}$$

$$S_0\,A_{ed}^{11\,+}(22\,+) = \frac{4\,\pi}{\sqrt{3}}\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)\,M_1^\gamma\,v_{LT}'\,. \tag{D.13}$$

(iii) Tensor asymmetries:

$$S_0\,A_d^{20\,+}(20\,+) = \frac{2\,\pi}{3}\,\Big((\sqrt{2}\,C_0^\gamma + C_2^\gamma)^2 + 2\,(C_2^\gamma)^2\Big)\,v_L - \frac{4\,\pi}{3}\,(M_1^\gamma)^2\,v_T\,, \tag{D.14}$$

$$S_0\,A_d^{20\,+}(21\,+) = -2\,\sqrt{2}\,\pi\,\Big(\sqrt{2}\,C_0^\gamma + C_2^\gamma\Big)\,M_1^\gamma\,v_{LT}\,, \tag{D.15}$$

$$S_0\,A_d^{20\,+}(22\,+) = -2\,\sqrt{\frac{2}{3}}\,\pi\,(M_1^\gamma)^2\,v_{TT}\,, \tag{D.16}$$

$$S_0\,A_d^{21\,+}(21\,+) = \frac{4\pi}{3}\Big(2\,(C_0^\gamma)^2 + \sqrt{2}\,C_0^\gamma C_2^\gamma - 2\,(C_2^\gamma)^2\Big)\,v_L - 2\,\pi\,(M_1^\gamma)^2\,(v_T - v_{TT})\,, \tag{D.17}$$

$$S_0\,A_d^{21\,+}(22\,+) = -\frac{4\,\pi}{\sqrt{3}}\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)\,M_1^\gamma\,v_{LT}\,, \tag{D.18}$$

$$S_0\,A_d^{22\,+}(22\,+) = \frac{4\,\pi}{3}\,\Big(\sqrt{2}\,C_0^\gamma - C_2^\gamma\Big)^2\,v_L\,. \tag{D.19}$$

(B) Asymmetries for P-violating contributions:
(i) Scalar asymmetries:

$$S_0\,A_d^{00\,+}(00\,+) = \frac{8\,\pi}{3}\,E_1^{Z_a^{\mathcal{A}}}\,M_1^\gamma\,v_T'\,, \tag{D.20}$$

$$S_0\,A_d^{00\,+}(10\,+) = 2\,\sqrt{\frac{2}{3}}\,\pi\,\Big(E_1^\gamma + E_1^{Z_a^{\mathcal{V}}}\Big)\,M_1^\gamma\,v_T + 2\,\sqrt{\frac{2}{3}}\,\pi\,M_1^\gamma\,M_1^{Z_v^{\mathcal{A}}}\,v_T'\,, \tag{D.21}$$

$$S_0\,A_d^{00\,+}(11\,+) = \frac{4\pi}{3}\Big(E_1^\gamma + E_1^{Z_a^{\mathcal{V}}}\Big)\,\Big(2\,\sqrt{2}\,C_0^\gamma + C_2^\gamma\Big)\,v_{LT} + \frac{2\,\sqrt{2}}{3}\,\pi\,\Big(\Big(4\,C_0^{Z_v^{\mathcal{A}}} + \sqrt{2}\,C_2^{Z_v^{\mathcal{A}}}\Big)\,M_1^\gamma + \Big(4\,C_0^\gamma + \sqrt{2}\,C_2^\gamma\Big)\,M_1^{Z_v^{\mathcal{A}}}\Big)\,v_{LT}'\,, \tag{D.22}$$

$$S_0\,A_d^{00\,+}(20\,+) = -\frac{2\,\sqrt{2}}{3}\,\pi\,E_1^{Z_a^{\mathcal{A}}}\,M_1^\gamma\,v_T'\,, \tag{D.23}$$

$$S_0\,A_d^{00\,+}(21\,+) = 4\,\pi\,C_2^\gamma\,E_1^{Z_a^{\mathcal{A}}}\,v_{LT}'\,, \tag{D.24}$$

$$S_0\,A^{00\,+}_{ed}(00+) = \frac{8\,\pi}{3}\left(C^\gamma_0\,C^{Z^{\mathcal{A}}_v}_0 + C^\gamma_2\,C^{Z^{\mathcal{A}}_v}_2\right)v_L$$
$$+\frac{8\,\pi}{3}\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,v_T + \frac{8\,\pi}{3}\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v'_T\,, \quad \text{(D.25)}$$

$$S_0\,A^{00\,+}_{ed}(10+) = 2\sqrt{\frac{2}{3}}\,\pi\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v_T\,, \quad \text{(D.26)}$$

$$S_0\,A^{00\,+}_{ed}(11+) = \frac{4\pi}{3}\left(2\sqrt{2}\,C^\gamma_0 + C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v_{LT}\,, \quad \text{(D.27)}$$

$$S_0\,A^{00\,+}_{ed}(20+) =$$
$$-\frac{4\,\pi}{3}\left(2\,C^{Z^{\mathcal{A}}_v}_0\,C^\gamma_2 + 2\,C^\gamma_0\,C^{Z^{\mathcal{A}}_v}_2 + \sqrt{2}\,C^\gamma_2\,C^{Z^{\mathcal{A}}_v}_2\right)v_L$$
$$-\frac{2\sqrt{2}}{3}\pi M^\gamma_1 M^{Z^{\mathcal{A}}_v}_1 v_T - \frac{2\sqrt{2}}{3}\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1 v'_T, \quad \text{(D.28)}$$

$$S_0\,A^{00\,+}_{ed}(21+) = 4\,\pi\left(C^{Z^{\mathcal{A}}_v}_2\,M^\gamma_1 + C^\gamma_2\,M^{Z^{\mathcal{A}}_v}_1\right)v_{LT}$$
$$+4\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)C^\gamma_2\,v'_{LT}, \quad \text{(D.29)}$$

$$S_0\,A^{00\,+}_{ed}(22+) = \frac{4\,\pi}{\sqrt{3}}\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,v_{TT}\,. \quad \text{(D.30)}$$

(ii) Vector asymmetries:

$$S_0\,A^{10\,+}_d(11+) = -2\sqrt{\frac{2}{3}}\,\pi\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v'_{LT}, \quad \text{(D.31)}$$

$$S_0\,A^{10\,+}_d(20+) =$$
$$\frac{4\,\pi}{\sqrt{3}}\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v_T + \frac{4\,\pi}{\sqrt{3}}\,M^{Z^{\mathcal{A}}_v}_1\,M^\gamma_1\,v'_T\,, \quad \text{(D.32)}$$

$$S_0\,A^{10\,+}_d(21+) = 2\sqrt{\frac{2}{3}}\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)v_{LT}$$
$$+2\sqrt{\frac{2}{3}}\,\pi\left(\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)M^\gamma_1\right.$$
$$\left.+\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)M^{Z^{\mathcal{A}}_v}_1\right)v'_{LT}\,, \quad \text{(D.33)}$$

$$S_0\,A^{11\,+}_d(11+) = 4\,\pi\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v'_T\,, \quad \text{(D.34)}$$

$$S_0\,A^{11\,+}_d(21+) =$$
$$4\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v_T + 4\,\pi\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,v'_T\,, \quad \text{(D.35)}$$

$$S_0\,A^{11\,+}_d(22+) = \frac{4\pi}{\sqrt{3}}\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)v_{LT}$$
$$+\frac{4\,\pi}{\sqrt{3}}\left(\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)M^\gamma_1\right.$$
$$\left.+\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)M^{Z^{\mathcal{A}}_v}_1\right)v'_{LT}\,, \quad \text{(D.36)}$$

$$S_0\,A^{10\,+}_{ed}(10+) =$$
$$\frac{4\,\pi}{3}\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)v_L\,, \quad \text{(D.37)}$$

$$S_0\,A^{10\,+}_{ed}(11+) = -2\sqrt{\frac{2}{3}}\,\pi\left(\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)M^\gamma_1\right.$$
$$\left.+\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)M^{Z^{\mathcal{A}}_v}_1\right)v_{LT}$$
$$-2\sqrt{\frac{2}{3}}\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)v'_{LT}\,, \quad \text{(D.38)}$$

$$S_0\,A^{10\,+}_{ed}(20+) = \frac{4\,\pi}{\sqrt{3}}\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v_T\,, \quad \text{(D.39)}$$

$$S_0\,A^{10\,+}_{ed}(21+) = 2\sqrt{\frac{2}{3}}\,\pi\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v_{LT}\,, \quad \text{(D.40)}$$

$$S_0\,A^{11\,+}_{ed}(11+) = \frac{4\,\pi}{3}\left(C^\gamma_0\left(4\,C^{Z^{\mathcal{A}}_v}_0 + \sqrt{2}\,C^{Z^{\mathcal{A}}_v}_2\right)\right.$$
$$\left.+C^\gamma_2\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - 4\,C^{Z^{\mathcal{A}}_v}_2\right)\right)v_L$$
$$+4\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v'_T$$
$$+4\,\pi\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,(v_T + v_{TT})\,, \quad \text{(D.41)}$$

$$S_0\,A^{11\,+}_{ed}(21+) = 4\,\pi\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v_T\,, \quad \text{(D.42)}$$

$$S_0\,A^{11\,+}_{ed}(22+) = \frac{4\,\pi}{\sqrt{3}}\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v_{LT}\,. \quad \text{(D.43)}$$

(iii) Tensor asymmetries:

$$S_0\,A^{20\,+}_d(20+) = -\frac{8\,\pi}{3}\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v'_T\,, \quad \text{(D.44)}$$

$$S_0\,A^{20\,+}_d(21+) = -2\,\pi\left(2\,C^\gamma_0 + \sqrt{2}\,C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v'_{LT}, \quad \text{(D.45)}$$

$$S_0\,A^{21\,+}_d(21+) = -4\,\pi\,E^{Z^{\mathcal{A}}_a}_1\,M^\gamma_1\,v'_T\,, \quad \text{(D.46)}$$

$$S_0\,A^{21\,+}_d(22+) = \frac{4\,\pi}{\sqrt{3}}\left(-\sqrt{2}\,C^\gamma_0 + C^\gamma_2\right)E^{Z^{\mathcal{A}}_a}_1\,v'_{LT}, \quad \text{(D.47)}$$

$$S_0\,A^{20\,+}_{ed}(20+) = \frac{4\,\pi}{3}\left(C^\gamma_0\left(2\,C^{Z^{\mathcal{A}}_v}_0 + \sqrt{2}\,C^{Z^{\mathcal{A}}_v}_2\right)\right.$$
$$\left.+C^\gamma_2\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 + 3\,C^{Z^{\mathcal{A}}_v}_2\right)\right)v_L - \frac{8\,\pi}{3}\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,v_T$$
$$-\frac{8\,\pi}{3}\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v'_T\,, \quad \text{(D.48)}$$

$$S_0\,A^{20\,+}_{ed}(21+) = -2\sqrt{2}\,\pi\left(\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 + C^{Z^{\mathcal{A}}_v}_2\right)M^\gamma_1\right.$$
$$\left.+\left(\sqrt{2}\,C^\gamma_0 + C^\gamma_2\right)M^{Z^{\mathcal{A}}_v}_1\right)v_{LT}$$
$$-2\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)\left(2\,C^\gamma_0 + \sqrt{2}\,C^\gamma_2\right)v'_{LT}\,, \quad \text{(D.49)}$$

$$S_0\,A^{20\,+}_{ed}(22+) = -4\sqrt{\frac{2}{3}}\,\pi\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,v_{TT}\,, \quad \text{(D.50)}$$

$$S_0\,A^{21\,+}_{ed}(21+) = \frac{4\,\pi}{3}\left(C^\gamma_0\left(4\,C^{Z^{\mathcal{A}}_v}_0 + \sqrt{2}\,C^{Z^{\mathcal{A}}_v}_2\right)\right.$$
$$\left.+C^\gamma_2\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - 4\,C^{Z^{\mathcal{A}}_v}_2\right)\right)v_L - 4\,\pi\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)M^\gamma_1\,v'_T$$
$$-4\,\pi\,M^\gamma_1\,M^{Z^{\mathcal{A}}_v}_1\,(v_T - v_{TT})\,, \quad \text{(D.51)}$$

$$S_0\,A^{21\,+}_{ed}(22+) = -\frac{4\,\pi}{\sqrt{3}}\left(\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)M^\gamma_1\right.$$
$$\left.+\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)M^{Z^{\mathcal{A}}_v}_1\right)v_{LT}$$
$$-\frac{4\,\pi}{\sqrt{3}}\left(E^\gamma_1 + E^{Z^{\mathcal{V}}_a}_1\right)\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)v'_{LT}\,, \quad \text{(D.52)}$$

$$S_0\,A^{22\,+}_{ed}(22+) =$$
$$\frac{8\,\pi}{3}\left(\sqrt{2}\,C^\gamma_0 - C^\gamma_2\right)\left(\sqrt{2}\,C^{Z^{\mathcal{A}}_v}_0 - C^{Z^{\mathcal{A}}_v}_2\right)v_L\,. \quad \text{(D.53)}$$

(C) Asymmetries for T-violating contributions:
(i) Scalar asymmetries:

$$S_0\, A_d^{00\,+}(11\,-) = 4\,\pi\, C_2^\gamma\, E_2^\gamma\, v_{LT}\,, \quad (D.54)$$

$$S_0\, A_{ed}^{00\,+}(21\,-) = \frac{4\,\pi}{3}\left(2\,\sqrt{2}\,C_0^\gamma + C_2^\gamma\right) E_2^\gamma\, v'_{LT}\,. \quad (D.55)$$

(ii) Vector asymmetries:

$$S_0\, A_d^{10\,+}(21\,-) = -2\sqrt{\frac{2}{3}}\,\pi\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma\, v_{LT}\,, \quad (D.56)$$

$$S_0\, A_d^{11\,+}(21\,-) = 4\,\pi\, E_2^\gamma\, M_1^\gamma\,(v_T + v_{TT})\,, \quad (D.57)$$

$$S_0\, A_d^{11\,+}(22\,-) = \frac{4\,\pi}{\sqrt{3}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma\, v_{LT}\,, \quad (D.58)$$

$$S_0\, A_{ed}^{10\,+}(11\,-) = 2\sqrt{\frac{2}{3}}\,\pi\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma\, v'_{LT}\,, \quad (D.59)$$

$$S_0\, A_{ed}^{11\,+}(11\,-) = 4\,\pi\, E_2^\gamma\, M_1^\gamma\, v'_T\,. \quad (D.60)$$

(iii) Tensor asymmetries:

$$S_0\, A_{ed}^{20\,+}(21\,-) = -\frac{2\pi}{3}\left(2\,C_0^\gamma + 5\,\sqrt{2}\,C_2^\gamma\right) E_2^\gamma\, v'_{LT}\,, \quad (D.61)$$

$$S_0\, A_{ed}^{21\,+}(21\,-) = -4\,\pi\, E_2^\gamma\, M_1^\gamma\, v'_T\,, \quad (D.62)$$

$$S_0\, A_{ed}^{21\,+}(22\,-) = -\frac{4\,\pi}{\sqrt{3}}\left(\sqrt{2}\,C_0^\gamma - C_2^\gamma\right) E_2^\gamma\, v'_{LT}\,. \quad (D.63)$$

Appendix E. Listing of nonvanishing asymmetries in terms of invariant form factors as in Appendix D

(A) Asymmetries for P- and T-conserved contributions:
(i) Scalar asymmetries:

$$S_0\, A_d^{00\,+}(00\,+) = G_C{}^2 + \frac{8}{9}\,\eta^2\, G_Q{}^2 + \frac{2}{3}\,\eta\left(1 + 2\,(1+\eta)\tan^2\frac{\theta}{2}\right) G_M{}^2\,, \quad (E.1)$$

$$S_0\, A_d^{00\,+}(20\,+) = -\frac{\eta}{3\,\sqrt{2}} \times\left(8(G_C + \frac{\eta}{3}G_Q)\,G_Q + (1 + 2\,(1+\eta)\tan^2\frac{\theta}{2})\,G_M{}^2\right), \quad (E.2)$$

$$S_0\, A_d^{00\,+}(21\,+) = \frac{4}{\sqrt{3}}\,\sec\frac{\theta}{2}\,\eta\,\sqrt{\eta\left(1 + \eta\sin^2\frac{\theta}{2}\right)}\, G_M\, G_Q\,, \quad (E.3)$$

$$S_0\, A_d^{00\,+}(22\,+) = -\frac{\eta}{\sqrt{3}}\, G_M{}^2\,, \quad (E.4)$$

$$S_0\, A_{ed}^{00\,+}(10\,+) = \sqrt{\frac{2}{3}}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_M{}^2\,, \quad (E.5)$$

$$S_0\, A_{ed}^{00\,+}(11\,+) = \frac{4}{\sqrt{3}}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_C + \frac{\eta}{3}\,G_Q\right) G_M\,. \quad (E.6)$$

(ii) Vector asymmetries:

$$S_0\, A_d^{10\,+}(10\,+) = \left(G_C - \frac{2}{3}\,\eta\, G_Q\right)^2, \quad (E.7)$$

$$S_0\, A_d^{10\,+}(11\,+) = -\sqrt{2}\sec\frac{\theta}{2}\,\sqrt{\eta\left(1 + \eta\sin^2\frac{\theta}{2}\right)} \times\left(G_C - \frac{2}{3}\,\eta\, G_Q\right) G_M\,, \quad (E.8)$$

$$S_0\, A_d^{11\,+}(11\,+) = 2\left((G_C + \frac{\eta}{3}\,G_Q)^2 - \eta^2\, G_Q^2\right) + 2\,\eta\,(1+\eta)\tan^2\frac{\theta}{2}\, G_M{}^2\,, \quad (E.9)$$

$$S_0\, A_{ed}^{10\,+}(20\,+) = \frac{2}{\sqrt{3}}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_M{}^2\,, \quad (E.10)$$

$$S_0\, A_{ed}^{10\,+}(21\,+) = \sqrt{2}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_C - \frac{2}{3}\,\eta\, G_Q\right) G_M\,, \quad (E.11)$$

$$S_0\, A_{ed}^{11\,+}(21\,+) = 2\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_M{}^2\,, \quad (E.12)$$

$$S_0\, A_{ed}^{11\,+}(22\,+) = 2\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_C - \frac{2}{3}\,\eta\, G_Q\right) G_M\,. \quad (E.13)$$

(iii) Tensor asymmetries:

$$S_0\, A_d^{20\,+}(20\,+) = \frac{1}{3}\left((G_C + 2\,\eta\, G_Q)^2 + 2\,G_C^2\right) - \frac{2}{3}\,\eta\left(1 + 2\,(1+\eta)\tan^2\frac{\theta}{2}\right) G_M{}^2\,, \quad (E.14)$$

$$S_0\, A_d^{20\,+}(21\,+) = -\sqrt{6}\,\sec\frac{\theta}{2}\,\sqrt{\eta\left(1 + \eta\sin^2\frac{\theta}{2}\right)} \times\left(G_C + \frac{2}{3}\,\eta\, G_Q\right) G_M\,, \quad (E.15)$$

$$S_0\, A_d^{20\,+}(22\,+) = \sqrt{\frac{2}{3}}\,\eta\, G_M{}^2\,, \quad (E.16)$$

$$S_0\, A_d^{21\,+}(21\,+) = 2\left((G_C + \frac{\eta}{3}\,G_Q)^2 - \eta^2\, G_Q^2\right) - 2\,\eta\left(1 + (1+\eta)\tan^2\frac{\theta}{2}\right) G_M{}^2\,, \quad (E.17)$$

$$S_0\, A_d^{21\,+}(22\,+) = -2\,\sec\frac{\theta}{2}\,\sqrt{\eta\left(1 + \eta\sin^2\frac{\theta}{2}\right)} \times\left(G_C - \frac{2}{3}\,\eta\, G_Q\right) G_M\,, \quad (E.18)$$

$$S_0\, A_d^{22\,+}(22\,+) = 2\left(G_C - \frac{2}{3}\,\eta\, G_Q\right)^2. \quad (E.19)$$

(B) Asymmetries for P-violating contributions:
(i) Scalar asymmetries:

$$S_0\,A_d^{00\,+}(00+) = \frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_{E1}^{Z_a^{\mathcal{A}}}\,G_M, \tag{E.20}$$

$$S_0\,A_d^{00\,+}(10+) = \sqrt{\frac{2}{3}}\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right) \times\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M + 2\sqrt{\frac{2}{3}}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)(1+\eta\,\sin^2\frac{\theta}{2})}\,G_M\,G_M^{Z_v^{\mathcal{A}}}, \tag{E.21}$$

$$S_0\,A_d^{00\,+}(11+) = \frac{4}{\sqrt{3}}\,\sec\frac{\theta}{2}\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)} \times\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C+\frac{\eta}{3}\,G_Q\right) +\frac{4}{\sqrt{3}}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\,\left(\left(G_C^{Z_v^{\mathcal{A}}}+\frac{\eta}{3}\,G_Q^{Z_v^{\mathcal{A}}}\right)G_M +\left(G_C+\frac{\eta}{3}\,G_Q\right)G_M^{Z_v^{\mathcal{A}}}\right), \tag{E.22}$$

$$S_0\,A_d^{00\,+}(20+) = -\frac{2\sqrt{2}}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_{E1}^{Z_a^{\mathcal{A}}}\,G_M, \tag{E.23}$$

$$S_0\,A_d^{00\,+}(21+) = \frac{4}{\sqrt{3}}\,\eta\sqrt{\eta\,(1+\eta)}\,\tan\frac{\theta}{2}\,G_{E1}^{Z_a^{\mathcal{A}}}G_Q, \tag{E.24}$$

$$S_0\,A_{ed}^{00\,+}(00+) = 2\,G_C\,G_C^{Z_v^{\mathcal{A}}} + \frac{16}{9}\,\eta^2\,G_Q^{Z_v^{\mathcal{A}}}\,G_Q +\frac{4}{3}\,\eta\left(1+2\,(1+\eta)\,\tan^2\frac{\theta}{2}\right)G_M\,G_M^{Z_v^{\mathcal{A}}} +\frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)} \times\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M, \tag{E.25}$$

$$S_0\,A_{ed}^{00\,+}(10+) = \sqrt{\frac{2}{3}}\,\eta \times\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right)G_{E1}^{Z_a^{\mathcal{A}}}\,G_M, \tag{E.26}$$

$$S_0\,A_{ed}^{00\,+}(11+) = \frac{4}{\sqrt{3}}\,G_{E1}^{Z_a^{\mathcal{A}}}\left(G_C+\frac{\eta}{3}\,G_Q\right) \times\sec\frac{\theta}{2}\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})}, \tag{E.27}$$

$$S_0\,A_{ed}^{00\,+}(20+) = -\frac{4\sqrt{2}}{3}\Big(G_C\,G_Q^{Z_v^{\mathcal{A}}}+G_C^{Z_v^{\mathcal{A}}}\,G_Q +\frac{2}{3}\,\eta\,G_Q^{Z_v^{\mathcal{A}}}\,G_Q\Big) - \frac{\sqrt{2}}{3}\,\eta\left(1+2\,(1+\eta)\,\tan^2\frac{\theta}{2}\right) \times G_M\,G_M^{Z_v^{\mathcal{A}}} - \frac{2\sqrt{2}}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M, \tag{E.28}$$

$$S_0\,A_{ed}^{00\,+}(21+) = \frac{4}{\sqrt{3}}\,\eta\left(G_Q^{Z_v^{\mathcal{A}}}\,G_M+G_M^{Z_v^{\mathcal{A}}}\,G_Q\right) \times\sec\frac{\theta}{2}\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})} +\frac{4}{\sqrt{3}}\,\eta\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_Q\,, \tag{E.29}$$

$$S_0\,A_{ed}^{00\,+}(22+) = -\frac{2}{\sqrt{3}}\,\eta\,G_M\,G_M^{Z_v^{\mathcal{A}}}\,. \tag{E.30}$$

(ii) Vector asymmetries:

$$S_0\,A_d^{10\,+}(11+) = -\sqrt{2}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\,G_{E1}^{Z_a^{\mathcal{A}}}\left(G_C-\frac{2}{3}\,\eta\,G_Q\right), \tag{E.31}$$

$$S_0\,A_d^{10\,+}(20+) = \frac{2}{\sqrt{3}}\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right)\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M +\frac{4}{\sqrt{3}}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\sqrt{(1+\eta)(1+\eta\,\sin^2\frac{\theta}{2})}\,G_M\,G_M^{Z_v^{\mathcal{A}}}, \tag{E.32}$$

$$S_0\,A_d^{10\,+}(21+) = \sqrt{2}\,\sec\frac{\theta}{2}\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)} \times\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C-\frac{2}{3}\,\eta\,G_Q\right)+\sqrt{2}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)} \times\left(\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\,\eta\,G_Q^{Z_v^{\mathcal{A}}}\right)G_M+\left(G_C-\frac{2}{3}\,\eta\,G_Q\right)G_M^{Z_v^{\mathcal{A}}}\right), \tag{E.33}$$

$$S_0\,A_d^{11\,+}(11+) = 4\,\sec\frac{\theta}{2}\,\eta\,\tan\frac{\theta}{2} \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\,G_{E1}^{Z_a^{\mathcal{A}}}\,G_M, \tag{E.34}$$

$$S_0\,A_d^{11\,+}(21+) = 2\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right)\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M +4\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\sqrt{(1+\eta)(1+\eta\,\sin^2\frac{\theta}{2})}\,G_M\,G_M^{Z_v^{\mathcal{A}}}\,, \tag{E.35}$$

$$S_0\,A_d^{11\,+}(22+) = 2\,\sec\frac{\theta}{2}\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)} \times\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C-\frac{2}{3}\,\eta\,G_Q\right) +2\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\,\eta\,G_Q^{Z_v^{\mathcal{A}}}\right)G_M +\left(G_C-\frac{2}{3}\,\eta\,G_Q\right)G_M^{Z_v^{\mathcal{A}}}\right), \tag{E.36}$$

$$S_0A_{ed}^{10\,+}(10+) = 2\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\eta\,G_Q^{Z_v^{\mathcal{A}}}\right)\left(G_C-\frac{2}{3}\eta\,G_Q\right), \tag{E.37}$$

$$S_0\,A_{ed}^{10\,+}(11+) = -\sqrt{2}\,\sec\frac{\theta}{2}\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})} \times\left(\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\,\eta\,G_Q^{Z_v^{\mathcal{A}}}\right)G_M+\left(G_C-\frac{2}{3}\,\eta\,G_Q\right)G_M^{Z_v^{\mathcal{A}}}\right) -\sqrt{2}\,\tan\frac{\theta}{2}\,\sqrt{\eta(1+\eta)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C-\frac{2}{3}\eta\,G_Q\right), \tag{E.38}$$

$$S_0\, A_{ed}^{10\,+}(20\,+) = \frac{2}{\sqrt{3}}\,\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right) G_{E1}^{Z_a^{\mathcal{A}}}\, G_M, \tag{E.39}$$

$$S_0\, A_{ed}^{10\,+}(21\,+) = \sqrt{2}\,\sec\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})}\, G_{E1}^{Z_a^{\mathcal{A}}}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right), \tag{E.40}$$

$$S_0\, A_{ed}^{11\,+}(11\,+) = 4\left(G_C^{Z_v^{\mathcal{A}}}\left(G_C+\frac{\eta}{3}\,G_Q\right)+\frac{\eta}{3}\,G_Q^{Z_v^{\mathcal{A}}}\left(G_C-\frac{8}{3}\,\eta\, G_Q\right)\right) +4\,\eta\,(1+\eta)\,\tan^2\frac{\theta}{2}\,G_M\, G_M^{Z_v^{\mathcal{A}}} + 4\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)G_M\,, \tag{E.41}$$

$$S_0\, A_{ed}^{11\,+}(21+) = 2\eta\left(1+2\,(1+\eta)\tan^2\frac{\theta}{2}\right) G_{E1}^{Z_a^{\mathcal{A}}}\, G_M, \tag{E.42}$$

$$S_0\, A_{ed}^{11\,+}(22\,+) = 2\,\sec\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right) G_{E1}^{Z_a^{\mathcal{A}}}\,. \tag{E.43}$$

(iii) Tensor asymmetries:

$$S_0\, A_{d}^{20\,+}(20\,+) = -\frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E1}^{Z_a^{\mathcal{A}}}\, G_M, \tag{E.44}$$

$$S_0\, A_{d}^{20\,+}(21\,+) = -\sqrt{6}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\, G_{E1}^{Z_a^{\mathcal{A}}}\left(G_C+\frac{2}{3}\,\eta\, G_Q\right), \tag{E.45}$$

$$S_0\, A_{d}^{21\,+}(21\,+) = -4\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E1}^{Z_a^{\mathcal{A}}}\, G_M, \tag{E.46}$$

$$S_0\, A_{d}^{21\,+}(22\,+) = -2\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right) G_{E1}^{Z_a^{\mathcal{A}}}\,, \tag{E.47}$$

$$S_0\, A_{ed}^{20\,+}(20\,+) = 2\left(G_C^{Z_v^{\mathcal{A}}}\left(G_C+\frac{2}{3}\,\eta\, G_Q\right)+\frac{2}{3}\,\eta\, G_Q^{Z_v^{\mathcal{A}}}\left(G_C+2\,\eta\, G_Q\right)\right) -\frac{4}{3}\,\eta\left(1+2\,(1+\eta)\,\tan^2\frac{\theta}{2}\right) G_M\, G_M^{Z_v^{\mathcal{A}}} - \frac{8}{3}\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)(1+\eta\,\sin^2\frac{\theta}{2})}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right) G_M\,, \tag{E.48}$$

$$S_0\, A_{ed}^{20\,+}(21\,+) = -\sqrt{6}\,\sec\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})} \times\left(\left(G_C^{Z_v^{\mathcal{A}}}+\frac{2}{3}\,\eta\, G_Q^{Z_v^{\mathcal{A}}}\right) G_M + \left(G_C\, G_M^{Z_v^{\mathcal{A}}}+\frac{2}{3}\,\eta\, G_Q\right) G_M^{Z_v^{\mathcal{A}}}\right) -\sqrt{6}\,\tan\frac{\theta}{2}\,\sqrt{\eta(1+\eta)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C+\frac{2}{3}\,\eta\, G_Q\right), \tag{E.49}$$

$$S_0\, A_{ed}^{20\,+}(22\,+) = 2\,\sqrt{\frac{2}{3}}\,\eta\, G_M\, G_M^{Z_v^{\mathcal{A}}}\,, \tag{E.50}$$

$$S_0\, A_{ed}^{21\,+}(21\,+) = 4\left(G_C^{Z_v^{\mathcal{A}}}\left(G_C+\frac{\eta}{3}\,G_Q\right)+\frac{\eta}{3}\,G_Q^{Z_v^{\mathcal{A}}}\left(G_C-\frac{8}{3}\,\eta\, G_Q\right)\right) -4\,\eta\left(1+(1+\eta)\,\tan^2\frac{\theta}{2}\right) G_M\, G_M^{Z_v^{\mathcal{A}}} +2\,\sqrt{\frac{2}{3}}\, G_M\, G_M^{Z_v^{\mathcal{A}}}\,\eta\, - 4\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta \times\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right) G_M, \tag{E.51}$$

$$S_0\, A_{ed}^{21\,+}(22\,+) = -2\,\sec\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta\,\sin^2\frac{\theta}{2})}\left(\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\,\eta\, G_Q^{Z_v^{\mathcal{A}}}\right) G_M +\left(G_C\, G_M^{Z_v^{\mathcal{A}}}-\frac{2}{3}\,\eta\, G_Q\right) G_M^{Z_v^{\mathcal{A}}}\right) - 2\,\tan\frac{\theta}{2} \times\sqrt{\eta\,(1+\eta)}\left(G_{E1}^{\gamma}+G_{E1}^{Z_a^{\mathcal{V}}}\right)\left(G_C-\frac{2}{3}\,\eta\, G_Q\right), \tag{E.52}$$

$$S_0 A_{ed}^{22\,+}(22+) = 4\left(G_C^{Z_v^{\mathcal{A}}}-\frac{2}{3}\eta\, G_Q^{Z_v^{\mathcal{A}}}\right)\left(G_C-\frac{2}{3}\eta\, G_Q\right). \tag{E.53}$$

(C) Asymmetries for T-violating contributions:

(i) Scalar asymmetries:

$$S_0\, A_{d}^{00\,+}(11\,-) = \frac{4}{\sqrt{3}}\,\sec\frac{\theta}{2}\,\eta\,\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E2}^{\gamma}\, G_Q\,, \tag{E.54}$$

$$S_0\, A_{ed}^{00\,+}(21\,-) = \frac{4}{\sqrt{3}}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\, G_{E2}^{\gamma}\left(G_C+\frac{1}{3}\,\eta\, G_Q\right). \tag{E.55}$$

(ii) Vector asymmetries:

$$S_0\, A_{d}^{10\,+}(21\,-) = -\sqrt{2}\,\sec\frac{\theta}{2}\,\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E2}^{\gamma}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right), \tag{E.56}$$

$$S_0\, A_{d}^{11\,+}(21\,-) = 4\eta\,(1+\eta)\,\tan^2\frac{\theta}{2}\, G_{E2}^{\gamma}\, G_M, \tag{E.57}$$

$$S_0\, A_{d}^{11\,+}(22\,-) = 2\,\sec\frac{\theta}{2}\,\sqrt{\eta\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E2}^{\gamma}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right), \tag{E.58}$$

$$S_0\, A_{ed}^{10\,+}(11\,-) = \sqrt{2}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\, G_{E2}^{\gamma}\left(G_C-\frac{2}{3}\,\eta\, G_Q\right), \tag{E.59}$$

$$S_0\, A_{ed}^{11\,+}(11\,-) = 4\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta}{2}\right)}\, G_{E2}^{\gamma}\, G_M. \tag{E.60}$$

(iii) Tensor asymmetries:

$$S_0\, A_{ed}^{20\,+}(21\,-) =$$

$$-\sqrt{\frac{2}{3}}\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\,G_{E2}^{\gamma}\,\Big(G_C + \frac{10}{3}\,\eta\,G_Q\Big), \quad \text{(E.61)}$$

$$S_0\, A_{ed}^{21\,+}(21\,-) =$$

$$-4\,\sec\frac{\theta}{2}\,\tan\frac{\theta}{2}\,\eta\,\sqrt{(1{+}\eta)\Big(1{+}\eta\sin^2\frac{\theta}{2}\Big)}\,G_{E2}^{\gamma}\,G_M, \quad \text{(E.62)}$$

$$S_0\, A_{ed}^{21\,+}(22\,-) =$$

$$-2\,\tan\frac{\theta}{2}\,\sqrt{\eta\,(1+\eta)}\,\Big(G_C - \frac{2\,\eta}{3}\,G_Q\Big)\,G_{E2}^{\gamma}\,. \quad \text{(E.63)}$$

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